Mathematics Grade 9 Curriculum

Implemented September 2010

New Nouveau Brunswick

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Curriculum Overview for K-9 Mathematics

BACKGROUND AND RATIONALE

Mathematics curriculum is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society.

It is essential the mathematics curriculum reflects current research in mathematics instruction. To achieve this goal, the Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in New Brunswick. The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the NCTM.

There is an emphasis in the New Brunswick curriculum on particular key concepts at each grade which will result in greater depth of understanding and ultimately stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

The intent of this document is to clearly communicate high expectations for students in mathematics education to all education partners. Because of the emphasis placed on key concepts at each grade level, time needs to be taken to ensure mastery of these concepts. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM Principles and Standards, 2000).

BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

The New Brunswick Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

- mathematics learning is an active and constructive process;
- learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates;
- learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort; and
- learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Goals for Mathematically Literate Students

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices. Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals. Striving toward success, and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Diverse Cultural Perspectives

Students come from a diversity of cultures, have a diversity of experiences and attend schools in a variety of settings including urban, rural and isolated communities. To address the diversity of knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students, a variety of teaching and assessment strategies is required in the classroom.

For example, studies have shown that Aboriginal students often have a whole-world view of the environment in which they live and learn best in a holistic way. This means that students look for connections in learning and learn best when mathematics is contextualized and not taught as discrete components. Traditionally, in Aboriginal culture learning takes place through active participation and little emphasis is placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is important that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized. The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks, 1993).

It is important to note that general instructional strategies appropriate for different learning styles for a given cultural or other group may not apply to all students from that group. It is also important to be aware that strategies that make learning more accessible for a given group will also apply to students beyond the target group. Teaching for diversity supports higher achievement in mathematics for all students.

Adapting to the Needs of All Learners

Teachers must adapt instruction to accommodate differences in student development as they enter school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom. The reality of individual student differences must not be ignored when making instructional decisions.

As well, teachers must understand and design instruction to accommodate differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

Connections Across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, and physical education.

ASSESSMENT

Ongoing, interactive assessment (*formative assessment*) is essential to effective teaching and learning. Research has shown that formative assessment practices produce significant and often substantial learning gains, close achievement gaps and build students' ability to learn new skills (Black & William, 1998, OECD, 2006). Student involvement in assessment promotes learning. Interactive assessment, and encouraging self-assessment, allows students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes:

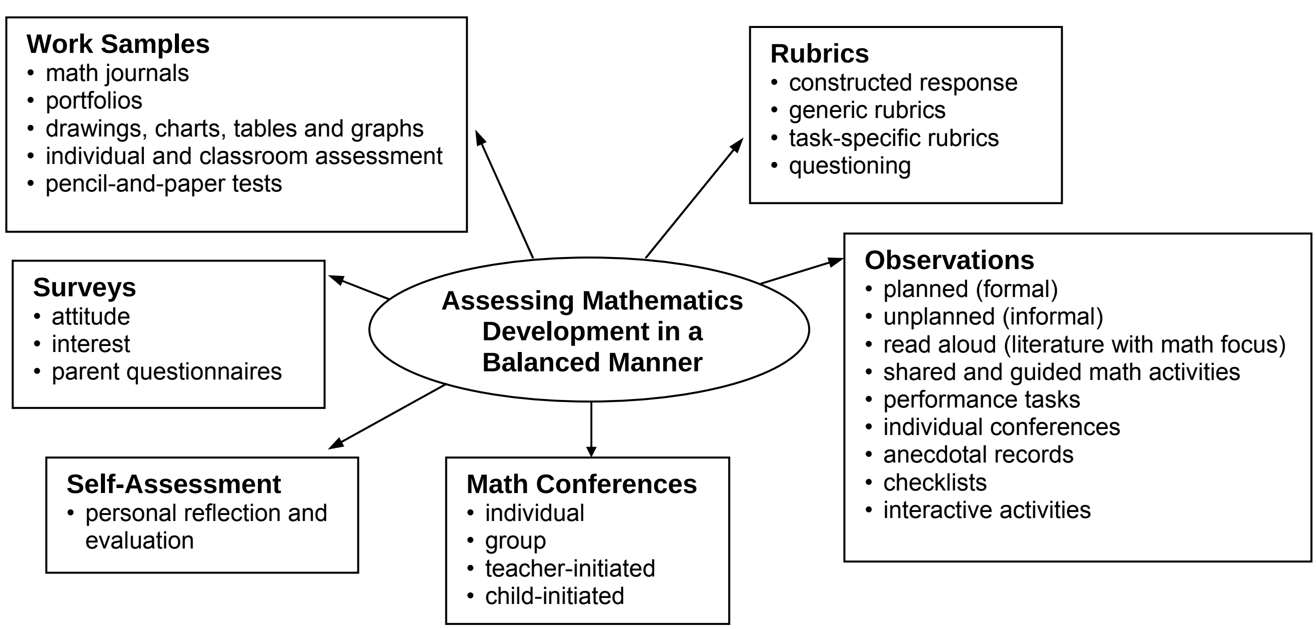
- providing clear goals, targets and learning outcomes
- using exemplars, rubrics and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning (Davies, 2000)

Formative assessment practices act as the scaffolding for learning which, only then, can be measured through summative assessment. *Summative assessment,* or assessment *of* learning, tracks student progress, informs instructional programming and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning and produce achievement gains.

Student assessment should:

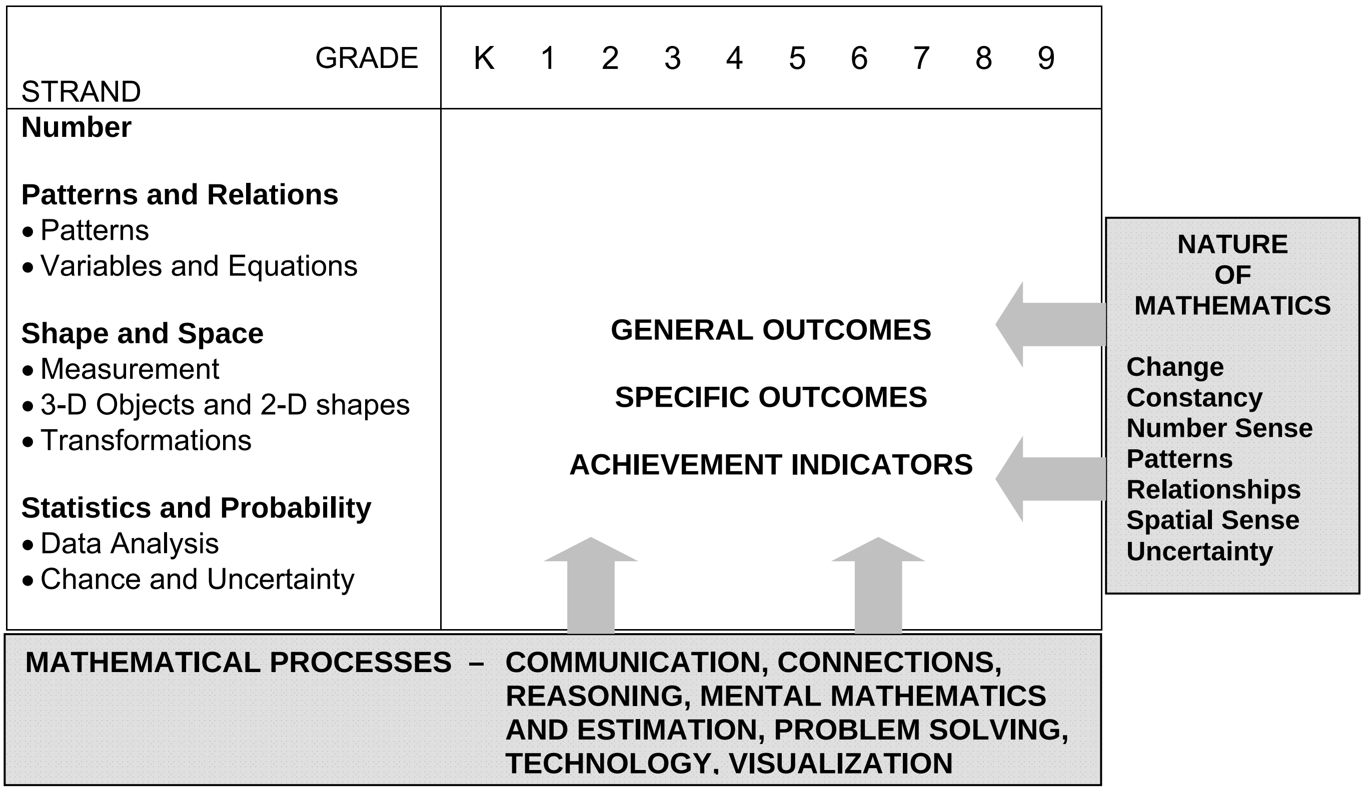
- align with curriculum outcomes
- use clear and helpful criteria
- promote student involvement in learning mathematics during and after the assessment experience
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction

(adapted from: NCTM, Mathematics Assessment: A practical handbook, 2001, p.22)



CONCEPTUAL FRAMEWORK FOR K–9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



INSTRUCTIONAL FOCUS

The New Brunswick K-9 Curriculum is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.
- There is a greater emphasis on mastery of specific curriculum outcomes.

The mathematics curriculum describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed. The components are not meant to stand alone. Activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between strands.

MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. Students are expected to:

- communicate in order to learn and express their understanding of mathematics (Communications: C)
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines (Connections: CN)
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation: ME)
- develop and apply new mathematical knowledge through problem solving (Problem Solving: PS)
- develop mathematical reasoning (Reasoning: R)
- select and use technologies as tools for learning and solving problems (Technology: T)
- develop visualization skills to assist in processing information, making connections and solving problems (Visualization: V).

The New Brunswick Curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"*Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*" (Caine and Caine, 1991, p. 5).

Reasoning [R]

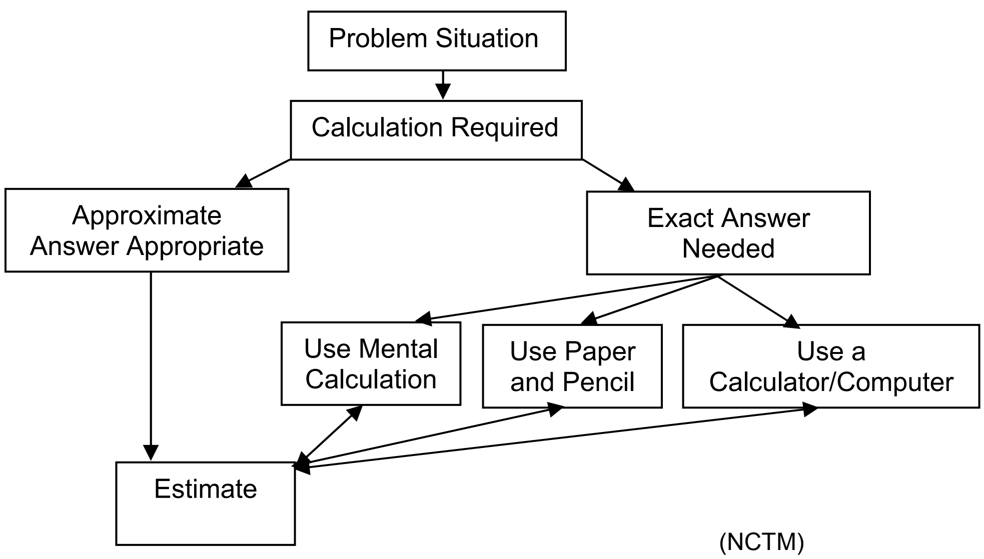
Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005).
Students proficient with mental mathematics "*become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving*" (Rubenstein, 2001). Mental mathematics "*provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers*" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision making process as described below.



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "*How would you...?*" or "*How could you...?*" the problem-solving approach is being modeled. Students develop their own problem-solving strategies by being open to listening, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K–3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization "*involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world*" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: **change**, **constancy**, **number sense**, **relationships**, **patterns**, **spatial sense** and **uncertainty**.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

STRUCTURE OF THE MATHEMATICS CURRICULUM

Strands

The learning outcomes in the New Brunswick Curriculum are organized into four strands across the grades, K–9. Strands are further subdivided into sub-strands which are the general curriculum outcomes.

Outcomes and Achievement Indicators

The New Brunswick Curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes and achievement indicators.

General Curriculum Outcomes (GCO) are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

Specific Curriculum Outcomes (SCO) are statements that identify specific concepts and related skills underpinned by the understanding and knowledge attained by students as required for a given grade.

Achievement Indicators are one example of a representative list of the depth, breadth and expectations for the outcome. Achievement indicators are pedagogy and context free.

Strand	General Curriculum Outcome (GCO)
Number (N)	**Number:** Develop number sense
Patterns and Relations (PR)	**Patterns**: Use patterns to describe the world and solve problems
	Variables and Equations: Represent algebraic expressions in multiple ways
Shape and Space (SS)	**Measurement**: Use direct and indirect measure to solve problems
	3-D Objects and 2-D Shapes: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them
	Transformations: Describe and analyze position and motion of objects and shapes
Statistics and Probability (SP)	**Data Analysis**: Collect, display and analyze data to solve problems
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty

CURRICULUM DOCUMENT FORMAT

This guide presents the mathematics curriculum by grade level so that a teacher may readily view the scope of the outcomes which students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings at a particular grade level are part of a bigger picture of concept and skill development.

As indicated earlier, the order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The specific curriculum outcomes (SCOs) are presented on individual four-page spreads as illustrated below.

GCO:
SCO: (specific curriculum outcome and mathematical processes)
Key for mathematical processes

Scope and Sequence
Current Grade

Elaboration
Guiding Questions

(Describes the "big ideas" and what the students should learn this year in regards to this concept.)

Page 1

GCO:
SCO:

Achievement Indicators

Guiding Questions

(Describes what could be observed to determine whether students have met the specific outcome.)

Page 2

GCO:
SCO:

Planning for Instruction
Guiding Questions

Choosing Instructional Strategies
(Lists general strategies to assist in teaching this outcome.)

Suggested Activities
(Lists possible specific activities to assist students in learning this concept.)

Possible Models

Page 3

GCO:
SCO:

Assessment Strategies
Guiding Questions

(Overview of assessment)

Whole Class/Group/Individual Assessment
(Lists sample assessment tasks.)

Follow-up on Assessment
Guiding Questions

Page 4

SCO: **N1: Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:** **• representing repeated multiplication using powers** **• using patterns to show that a power with an exponent of zero is equal to one** **• solving problems involving powers.** **[C, CN, PS, R]**
[C] Communication **[T]** Technology **[PS]** Problem Solving **[V]** Visualization **[CN]** Connections **[R]** Reasoning **[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
N1 Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers).	**N1** Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by: • representing repeated multiplication using powers • using patterns to show that a power with an exponent of zero is equal to one • solving problems involving powers.	**AN3** ***Number, Relations and Functions 10*** Demonstrate an understanding of powers with integral and rational exponents

ELABORATION

Guiding Questions:
• What do I want my students to learn?
• What do I want my students to understand and be able to do?

Students have had experience with perfect squares in relation to calculating area in grade 8. The terms **exponent**, **base,** and **power** (an expression made up of an exponent and a base) are used differently in different resources. For example, the power 6^4 (where 6 is the base and 4 is the exponent), may be described as "six to the power of four", "the fourth power of six", or as "six raised to the power of four" in various textbooks. For consistency and understanding, teachers are asked to use "*six to the exponent of four*", or "*six to the fourth*" in lieu of "*six to the power of four*".

Students should be able to link the term "squared" with a 2-D area model and "cubed" with the 3-D volume model. This will help connect units for area and volume (e.g., square centimetres as cm^2, cubic metres at m^3) to measurement and geometry. It should be emphasized that sometimes the same number can be expressed in multiple ways using powers (e.g. 64 = 8^2 or 4^3 or 2^6).

Students should be able to express 3^5 as $3 \times 3 \times 3 \times 3 \times 3$ and $5 \times 5 \times 5$ as 5^3. Students should also be able to explain the role of parentheses in powers by evaluating a given set of powers. For example:

$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$, where the base is -2.

$-(2^4) = -(2 \times 2 \times 2 \times 2) = -16$, where the base is 2.

$-2^4 = -(2 \times 2 \times 2 \times 2) = -16$, where the base is 2.

Students should be able to demonstrate that $a^0 = 1, a \neq 0$ for a given value of a, using patterns.

For example:

10^3	$=1000$
10^2	$= 100$
10^1	$= 10$
10^0	$= 1$

3^3	$= 27$
3^2	$= 9$
3^1	$= 3$
3^0	$= 1$

Alternatively:
$\frac{10^2}{10^2} = \frac{100}{100} = 1$ OR $\frac{10^2}{10^2} = 10^{2-2} = 10^0 = 1$
So… $\frac{a^2}{a^2} = a^{2-2} = a^0 = 1$

Students should be given opportunities to solve problems within a real world context.

SCO**: N1: Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:**
- **representing repeated multiplication using powers**
- **using patterns to show that a power with an exponent of zero is equal to one**
- **solving problems involving powers.**

[C, CN, PS, R]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Demonstrate the differences between the exponent and the base by building models of a given power, such as 2^3 and 3^2.
- Express a given power as a repeated multiplication.
- Express a given repeated multiplication as a power.
- Explain, using repeated multiplication, the difference between two given powers in which the exponent and base are interchanged, e.g. 10^3 and 3^{10}.
- Evaluate powers with integral bases (excluding base 0) and whole number exponents, e.g. $(-3)^4$, 2^3, -5^3, -10^0, 23^0.
- Explain the role of parentheses in powers by evaluating a given set of powers, e.g., compare $(-2)^4$, $-(2^4)$, -2^4, (-2^4).
- Demonstrate, using patterns, that a^0 is equal to 1 for a given value of a ($a \neq 0$).

SCO**: N1: Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:**
- **representing repeated multiplication using powers**
- **using patterns to show that a power with an exponent of zero is equal to one**
- **solving problems involving powers.**

[C, CN, PS, R]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:
- Provide students with many opportunities to explore concrete and pictorial representations of 2-D and 3-D models in order to differentiate between the base and the exponent.
- Investigate the difference between pairs of powers such as: 6^2 and 2^6, 5^8 and 8^5.
- Investigate the pattern in powers using a given base and exponents from 4 to 0 such as: $3^4, 3^3, 3^2, 3^1, 3^0$.
- Provide students with the opportunity to explore the difference between negative and positive bases, with or without parentheses.
- Explore the calculator to find the most efficient way to evaluate powers (e.g. y^x).

Suggested Activities

1. Provide 25 tiles and 30 cube-a-links. Have students explore the number of tiles required to make squares and the number of cube-a-links required to make cubes. Students should investigate squares with sides of 1, 2, 3, 4 and 5 tiles. They should also investigate cubes with sides of 1, 2, and 3 cube-a-links, while a cube with a side of 4 cube-a-links could be done as a whole group.

2. Have students copy and complete the following table.

Repeated Multiplication	Power	Value
a)	6^3	
b) $3 \times 3 \times 3 \times 3 \times 3 \times 3$		
c)		64
d)	$(-3)^4$	
e) $-(5 \times 5 \times 5)$		
f)		-49

Possible Models: colour tiles, linking cubes

SCO: **N1: Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:**
- **representing repeated multiplication using powers**
- **using patterns to show that a power with an exponent of zero is equal to one**
- **solving problems involving powers.**

[C, CN, PS, R]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. Demonstrate using patterns, that $8^0 = 1$.
2. Express 25 as a power where the exponent is 2 and the base is:
   a) positive
   b) negative.
3. Explain why 6^2 is called a square number while 6^3 is called a cube number.
4. Determine if $(-6)^2 = -6^2$. Explain why or why not. Determine if the same reasoning applies to $(-6)^3 = -6^3$.
5. Identify the base in the expression $-(-7)^5$.
6. Write as a power: $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$.
7. Write 32 as a power with a base of 2.
8. If three students with cold germs on their hands each shook hands with three other students, and those students each shook hands with three other students, how many students would be exposed to the cold germs?

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N2: Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents. [C, CN, PS, R, T]**			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
	N2 Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents.	**AN3** ***Number, Relations and Functions 10*** Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

The primary focus at grade 9 should be to develop an understanding of the **exponent laws** involving powers with integral bases (excluding base 0) and whole number exponents. Instruction should be designed so that students have the opportunity to discover rules and relationships and are able to confirm their discoveries. Emphasis on attaching names to the laws is not the focus.

When questions involve the sum and difference of powers, order of operations should be emphasized (e.g. $6^5 + 6^2 \neq 6^7$). In the simplification of expressions involving powers, students should be able to identify and explain error(s) e.g. $(2^3)^2 \neq 2^5$ or $(5^3) \times (5^4) \neq (5)^{12}$.

Expressions should be simplified as much as possible before they are evaluated, and before calculators are used. At this level, practice should involve numerical bases only (extensions to literal bases will be addressed in grade 10).

The chart below shows the relationship between repeated multiplication and exponent laws using repeated multiplication. Students who have an understanding of the relationship between all three components of this table will be able to manipulate numbers to solve problems using various strategies.

Sample Question	Repeated Multiplication	Simplification
$(-3)^2 \times (-3)^5$	$[(-3) \times (-3)] \times [(-3) \times (-3) \times (-3) \times (-3) \times (-3)]$	$(-3)^{2+5} = (-3)^7$
$\frac{(-3)^5}{(-3)^2}$	$\frac{(-3) \times (-3) \times (-3) \times (-3) \times (-3)}{(-3) \times (-3)}$	$(-3)^{5-2} = (-3)^3$
$[(-3)^2]^5$	$[(-3) \times (-3)] \times [(-3) \times (-3)] \times [(-3) \times (-3)] \times [(-3) \times (-3)] \times [(-3) \times (-3)]$	$(-3)^{2\times5} = (-3)^{10}$
$[(-3) \times 4]^2$	$[(-3) \times 4] \times [(-3) \times 4]$	$(-3)^2 \times 4^2$
$\left(\frac{-3}{4}\right)^3$	$\left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right)$	$\frac{(-3)^3}{4^3}$

*** This concept is taught in preparation for literal bases addressed in grade 10*

SCO: **N2: Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents.**
[C, CN, PS, R, T]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Explain, using examples, the exponent laws of powers with integral bases (excluding base 0) and whole number exponents:

  $(a^m)(a^n) = a^{m+n}$

  $a^m \div a^n = a^{m-n}, m > n$

  $(a^m)^n = a^{mn}$

  $(ab)^m = a^m b^m$

  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$

- Evaluate a given expression by first applying the exponent laws.
- Determine the sum of two given powers, e.g. $5^2 + 5^3$, and record the process.
- Determine the difference of two given powers, e.g. $4^3 - 4^2$, and record the process.
- Identify the error(s) in a given simplification of an expression involving powers

SCO: **N2: Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents.**
[C, CN, PS, R, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Ensure that there is a clear understanding of powers as a short way to represent a repeated multiplication of the same number.
- Have students show the relationship between repeated multiplication and exponent laws using repeated multiplication (see *Elaboration* for chart illustrating this relationship). This concept is taught in preparation for literal bases addressed in grade 10.
- Place an emphasis on simplification of expressions using exponent laws before evaluation.
- Note special cases e.g. when there is no exponent a "1" is understood: $5 = 5^1$.
- Have students demonstrate an understanding of exponent laws through explanations of inappropriate use of exponent laws e.g. $(2+3^2)^3 \neq 2^3 + 3^6$.

Suggested Activities

Solve the following:

1. Explain how to write a product or a quotient of powers as a single power.
2. $(-5)^3 \times (-5)^7$ $\left(\frac{2}{5}\right)^5$ $[(-3)^2]^3$ $[5 \times (-4)]^3$
3. The "9" key on your calculator has fallen off. Explain how you can find the value of 9^4 without using the "9" key.
4. Explain two ways that $[(-4) \times 5]^3$ can be calculated. Which way is more efficient?
5. Solve $7^9 \div (7^7 \times 7^1)$ mentally.
6. Simplify the following expression by first rearranging the powers and putting like bases together. $2^4 \times 5^3 \times 2^6 \times 10^2 \times 10^3 \times 5^{10}$.
7. Express $2^4 \times 3^4$ with a single base using the law of exponents. Extension: Simplify $8^2 \times 2^5$ to a single base using the law of exponents. Hint: first rewrite the expression so that both terms have the same base

Possible Models:

SCO: **N2: Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents.**
[C, CN, PS, R, T]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. Explain why $(-5) \times (-5)^6 \times (-5)^2 = (-5)^9$.
2. Show why $(4^2)^5 = 4^{10}$.
3. Write the expression $6^5 \times 5^5$ using only one exponent.
4. Write the expression $\frac{4^4 \times 4}{4^2}$ in simplified form, and then evaluate.
5. Write the following expression as the division of two powers. $\left(\frac{-4}{7}\right)^4$
6. Evaluate:

   a) $-(-3)^5$ b) $(1-3)^4 \div 2^2$
7. The prime factorization of 1024 is $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$. Write 1024 as the product of two powers of 2 in as many ways as possible.
8. Yvan made an error in simplifying the following expression:

   $(15 \div 5)^4 + (2+5)^2$

   $= (3)^4 + 2^2 + 5^2$

   $= 81 + 4 + 25 \;= 110$

   a) Indicate where he made his mistake and describe it.

   b) Show the correct procedure and determine the correct answer.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO**: N3: Demonstrate an understanding of rational numbers by:** • **comparing and ordering rational numbers** • **solving problems that involve arithmetic operations on rational numbers.** **[C, CN, PS, R, T, V]**
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Math and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
N4 Demonstrate an understanding of ratio and rate. **N5** Solve problems that involve rates, ratios and proportional reasoning. **N6** Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.	**N3** Demonstrate an understanding of rational numbers by: • comparing and ordering rational numbers • solving problems that involve arithmetic operations on rational numbers.	**AN2** ***Number, Relations and Functions 10*** Demonstrate an understanding of irrational numbers by representing, identifying and simplifying irrational numbers; ordering irrational numbers. **N1** ***Geometry, Measurement and Finance 10*** Solve problems that involve unit pricing and currency exchange, using proportional reasoning. **N2** ***Geometry, Measurement and Finance 10*** Demonstrate an understanding of income, including wages, salary, contracts, commissions, and piecework, to calculate gross pay and net pay.

ELABORATION

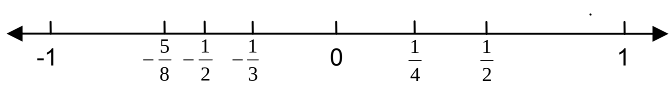
Guiding Questions:

• *What do I want my students to learn?*
• *What do I want my students to understand and be able to do?*

A **rational number** is any number that can be written as a fraction or a ratio of two integers $\frac{a}{b}$, where b is never zero. Students had experience with ratios, integers, positive decimal and fraction operations in middle school. Negative fraction operations will be introduced in grade 9. A review of integer and fraction operations will be necessary. The placement of a negative sign in a fraction will be an extension of what students have learned in the past. It is important for students to understand that: $\frac{6}{-2}$, $\frac{-6}{2}$ and $-\frac{6}{2}$ are all equivalent fractions. This becomes apparent when the division is completed and all fractions equal – 3, regardless of where the negative sign is placed.

Comparing and ordering rational numbers largely draws upon students' number sense. Strategies for ordering numbers should include the following:

- Understanding that a negative number is always less than a positive number.
- Developing a number line with zero marking the switch from positive to negative numbers, and with positioning of positive and negative benchmark fractions without conversion to decimals:



  - Comparing fractions with the same denominator, with unlike denominators and with the same numerator. Students should develop a variety of strategies to compare fractions in addition to finding common denominators.
  - Identifying fractions between any two given fractions, or decimals between any two decimals such as: 0.3 and 0.4, $\frac{1}{2}$ and $\frac{1}{3}$, $-\frac{1}{2}$ and $-\frac{1}{3}$.
- Reading fractions properly ($-\frac{1}{8}$ is read as negative one-eighth)
- Establishing that the further a negative number is from zero, the smaller it becomes.

SCO: **N3: Demonstrate an understanding of rational numbers by:**
- **comparing and ordering rational numbers**
- **solving problems that involve arithmetic operations on rational numbers.**

[C, CN, PS, R, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Order a given set of rational numbers, in fraction and decimal form, by placing them on a number line, e.g. $\frac{3}{5},\ -0.666\ldots,\ 0.5,\ -\frac{5}{8}$
- Identify a rational number that is between two given rational numbers.
- Solve a given expression involving operations on rational numbers in fraction form and decimal form.

SCO: **N3: Demonstrate an understanding of rational numbers by:**
- **comparing and ordering rational numbers**
- **solving problems that involve arithmetic operations on rational numbers.**

[C, CN, PS, R, T, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Generalize and apply a rule for determining the sign of the product or quotient of rational numbers through exploring patterns.
- Present examples of positive values with concrete models and pictures before moving to symbolic representation or introducing negative values.
- Extend the common denominator method for division of fractions, taught in grade 8, to negative fractions. When denominators are the same, the numerators can be divided as in the following example:

  $$\frac{5}{3} \div \frac{-1}{2} = \frac{10}{6} \div \frac{-3}{6} = \frac{10 \div (-3)}{6 \div 6} = \frac{\left(-\frac{10}{3}\right)}{1} = -\frac{10}{3}$$

  The answer, read as “negative ten thirds”, can be left as is unless the context of the question requires it to be expressed in the mixed form: $-3\frac{1}{3}$.
- Compare the multiplication and division of fractions using the meanings of the operations in expressions such as:

  $8 \div -\frac{1}{2} = -16$ *(How many halves in 8? How does the negative sign affect the answer?)*

  $8 \times -\frac{1}{2} = -4$ *(What is half of 8? How does the negative sign affect the answer?)*
- Use number lines as models for comparing and ordering rational numbers and for addition and subtraction of rational numbers.
- For this outcome, calculators should be discouraged as careful attention should be given to the numbers used.

Suggested Activities

1. Use patterning to justify the result for a negative multiplied by a negative using rational numbers. Have students complete the following pattern:

$3 \times \frac{-1}{2} = -\frac{3}{2}$ $\quad 2 \times \frac{-1}{2} = -1$ $\quad 1 \times \frac{-1}{2} = -\frac{1}{2}$ $\quad 0 \times \frac{-1}{2} = 0$ $\quad -1 \times \frac{-1}{2} = \frac{1}{2}$ $\quad -2 \times \frac{-1}{2} = \square$ $\quad -3 \times \frac{-1}{2} = \square$

Possible Models: number lines, fraction pieces, integer tiles

SCO**: N3: Demonstrate an understanding of rational numbers by:**
- **comparing and ordering rational numbers**
- **solving problems that involve arithmetic operations on rational numbers.**

[C, CN, PS, R, T, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. Use estimation to determine which expression has the greatest quotient.

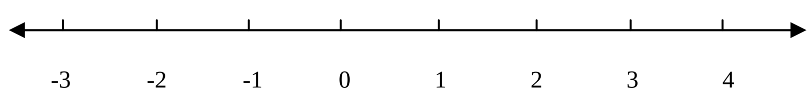
   $\frac{9}{5} \div \frac{3}{3}$ $\quad 2\frac{1}{5} \div 1\frac{6}{8}$ $\quad -3\frac{1}{10} \div \frac{5}{6}$ $\quad -\frac{1}{4} \div -\frac{1}{2}$

2. Find three rational numbers which lie between each of the following:

   $-1 \text{ and } 0$ $\quad \frac{1}{2} \text{ and } \frac{1}{3}$ $\quad -3.5 \text{ and } -3.6$ $\quad -\frac{1}{3} \text{ and } -0.4$ $\quad -\frac{2}{3} \text{ and } -0.6$

3. Order the following rational numbers on the number line:

   2.6 $\quad -\frac{1}{2}$ $\quad \frac{5}{3}$ $\quad 1.2$ $\quad -\frac{7}{8}$



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N4: Explain and apply the order of operations, including exponents, with and without technology.**
[PS, T]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
	N4 Explain and apply the order of operations, including exponents, with and without technology.	**AN2** ***Number, Relations and Functions 10*** Demonstrate an understanding of irrational numbers by representing, identifying, simplifying and ordering irrational numbers. **N1** ***Geometry, Measurement and Finance 10*** Solve problems that involve unit pricing and currency exchange, using proportional reasoning. **N2** ***Geometry, Measurement and Finance 10*** Demonstrate an understanding of income, including wages, salary, contracts, commissions, and piecework to calculate gross pay and net pay.

ELABORATION

Guiding Questions:
• What do I want my students to learn?
• What do I want my students to understand and be able to do?

Order of operations was introduced in grade 6 and practiced in grade 7 and 8 when solving problems involving a variety of operations with integers, positive decimals and fractions. In grade 9, they will extend the rules of order of operations to exponents and negative rational numbers.

The order of operations is:
1. Brackets
2. Exponents
3. Multiplication and division from left to right
4. Addition and subtraction from left to right

It is important for students to demonstrate their understanding of the order of operations, with and without the use of calculators. Students should demonstrate a competence in evaluating expressions that include fractions, fractions squared or cubed, decimals, and negative integers.

Students can use calculators however, be aware that entry sequences of calculators will vary. An exploration of this variation could offer an opportunity to develop a better understanding of order of operations. It is important for students to know how their personal calculators process the input and that they are able to apply this knowledge.

As an indication of understanding, students should be given steps towards an incorrect solution to a problem and be able to identify the step at which the error occurred.

SCO: **N4: Explain and apply the order of operations, including exponents, with and without technology.**
[PS, T]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Solve a given problem by applying the order of operations without the use of technology.
- Solve a given problem by applying the order of operations with the use of technology.
- Identify the error in applying the order of operations in a given incorrect solution.

SCO: **N4: Explain and apply the order of operations, including exponents, with and without technology.**
[PS, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

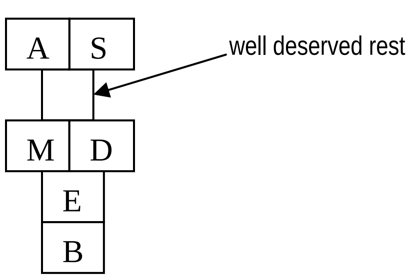
- Explore a variety of expressions in which brackets, fractions and negative numbers are used. Through these explorations students will demonstrate that the rules for order of operations ensure a consistent result.
- Have students compare their results when using calculators to simplify expressions and, if different from one another, determine how different calculators interpret data inputs differently. This could be an opportunity to establish the importance of order of operations, and the importance of correct calculator use.

Suggested Activities

1. Simplify expressions which include fractions and a mix of operations, without the use of a calculator, and express the answer as a fraction. Explore how the insertion of one or more set of brackets at various positions affects the answer. For example:

$\frac{1}{2}\times\frac{2}{3}-\frac{4}{3}\times\frac{3}{2}=-\mathbf{\frac{5}{3}}$ $\quad\frac{1}{2}\times\left(\frac{2}{3}-\frac{4}{3}\right)\times\frac{3}{2}=-\mathbf{\frac{1}{2}}$ $\quad\left(\frac{1}{2}\times\frac{2}{3}-\frac{4}{3}\right)\times\frac{3}{2}=-\mathbf{\frac{3}{2}}$ $\quad\frac{1}{2}\times\left[\left(\frac{2}{3}-\frac{4}{3}\right)\times\frac{3}{2}\right]=-\mathbf{\frac{1}{2}}$

2. Use a calculator to explore the use of brackets to simplify expressions with multiple terms in the numerator and denominator.
3. You have been hired by a company to produce a skill-testing math question using order of operations. Create a question with the solution, to be used to determine the prize winner.
4. Explain why the following hopscotch analogy works with respect to order of operations.



Possible Models: calculator

SCO: **N4: Explain and apply the order of operations, including exponents, with and without technology.**
[PS, T]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. $\frac{1}{2} - \frac{1}{3} \times \frac{1}{2} - \frac{1}{3}$
   Without the use of a calculator, simplify the expression and express your answer as a fraction.
   By inserting one pair of brackets, how many different answers are possible?
   By inserting two pairs of brackets, is it possible to receive a different answer?
2. Use a calculator to simplify the following: $\frac{56.3-22.5}{4.2\times(5.9-10.5)}$
3. Solve: $\frac{1}{3} - \frac{1}{2}$
4. Arrange the solutions of the following from least to greatest:
   $a)\ \frac{-3}{4} - \left(-\frac{3}{4} + \frac{4}{-5}\right)$ $\quad b)\ \frac{-3}{5} - \frac{-3}{4} - \frac{9}{-10}$ $\quad c)\ 6 \div \frac{-1}{5} - \frac{1}{-2}$ $\quad d)\ \frac{3}{5} - \left(\frac{-3}{5} - \frac{-2}{3}\right)$
5. Indicate at which step an error occurred and explain:
   $5 - 2(4+5)^2$
   $5 - 2(9)^2$ $\quad Step\ 1$
   $3(9)^2$ $\quad Step\ 2$
   $3(81)$ $\quad Step\ 3$
   243 $\quad Step\ 4$.
6. Use a calculator to convert the following Fahrenheit temperatures to Celsius, using the given formula: $°C = \frac{5}{9}(°F - 32°)$
   $a)\ 10°F$ $\quad b)\ 15°F$ $\quad c)\ -17.2°F$
7. Simplify the following: $\dfrac{\frac{1}{5}+\frac{3}{10}}{\frac{-1}{4}-1\frac{2}{5}} \times (-2) \div 5$

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N5: Determine the square root of positive rational numbers that are perfect squares. [C, CN, PS, R, T]**			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
N1 Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). **N2** Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).	**N5** Determine the square root of positive rational numbers that are perfect squares.	**AN1** ***Number, Relations and Functions 10*** Demonstrate an understanding of factors of whole numbers by determining the prime factors; greatest common factor; least common multiple; square root; cube root.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students in Grade 8 were exposed to square roots of whole numbers up to $\sqrt{144}$, including both perfect squares and estimates of non-perfect squares. They would have seen various models of perfect squares, such as square shapes drawn on grid paper or constructed with color tiles. They would have found square roots of perfect squares by prime factorization, mental computation, estimation and using the calculator. These strategies should be revisited accompanied by a discussion about when to use which strategy.

In Grade 9, the study of square roots is extended to finding the square root of positive rational numbers that are perfect squares, including whole numbers, fractions and decimals. Mathematicians use √ to represent only positive roots, so the solution to $\sqrt{25}$, is 5, which is called the principal square root. However, when solving an equation such as $x^2 = 4$, there are two solutions, $+2$ and -2:

$$x^2 = 4$$
$$x = \pm\sqrt{4}$$
$$x = \pm 2$$

Students should learn whole number perfect squares to 400 and be able to determine perfect squares beyond 400 through guess and test, using estimation strategies and/or prime factorization. For example if a student knows that $\sqrt{144} = 12$ and that $\sqrt{400} = 20$, they could estimate that $\sqrt{256}$ lies somewhere between 12 and 20.

Fraction and decimal square roots will all be variations of whole number perfect squares. For example, students will be asked to find $\sqrt{\frac{36}{25}}$, $\sqrt{0.25}$, $\sqrt{1.44}$. Students should also be able to explain why 25 and 0.25 are perfect squares but 2.5 is not.

Students should be able to determine a number given its square root. For example, the square root of a number is 0.7. What is the number? This relates to the fact that squares and square roots are inverse operations which should be explored. If you find the square root of a number and then square it, you end up where you started.

SCO: **N5: Determine the square root of positive rational numbers that are perfect squares. [C, CN, PS, R, T]**

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Determine whether or not a given rational number is a square number and explain the reasoning.
- Determine the square root of a given positive rational number that is a perfect square.
- Identify the error made in a given calculation of a square root, e.g., Is 0.8 the square root of 6.4?
- Determine a positive rational number given the square root of that positive rational number.

SCO: **N5: Determine the square root of positive rational numbers that are perfect squares. [C, CN, PS, R, T]**

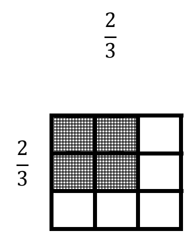
PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies



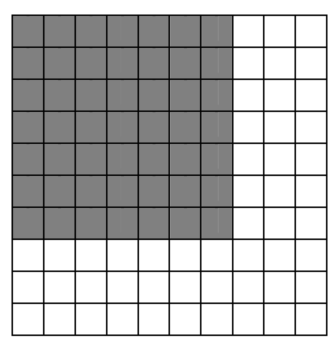
Consider the following strategies when planning lessons:

- Use area models to explore square root for fractions, just as they have been for whole numbers. For example, the diagram shows a square (representing the whole or 1) with $\frac{4}{9}$ shaded. The square root of $\frac{4}{9}$ is determined by finding the dimensions of the shaded square: $\frac{2}{3}$.

Suggested Activities

1. Relate finding the square of a number to squaring the length of the sides of a square. Relate finding the dimensions of a square to finding the square root of a number.
2. Challenge students to determine $\sqrt{\frac{8}{18}}$. What would be the first step?

   Note that all equivalent fractions would have the same answer: $\sqrt{\frac{4}{9}} = \sqrt{\frac{8}{18}} = \sqrt{\frac{12}{27}} = \frac{2}{3}$.
3. Have students investigate whether the square root of numbers greater than 1 is always less than the original number. For example, $\sqrt{64} = 8$, $\sqrt{1.21} = 1.1$ etc. This can lead to the misconception that this is always true for all numbers. Have students look at the square roots of perfect squares that are less than 1, like $\sqrt{\frac{4}{9}} = \frac{2}{3}$ or $\sqrt{0.01} = 0.1$. The area model can be used for decimals using a hundredths grid. For example, $\sqrt{0.49}$ is modeled here:



4. Connect the square roots of decimals to those of fractions. For example, $\sqrt{0.25} = \sqrt{\frac{25}{100}}$

Possible Models: grid paper, geoboards. colour tiles, calculators

SCO: **N5: Determine the square root of positive rational numbers that are perfect squares.**
[C, CN, PS, R, T]

ASSESSMENT STRATEGIES

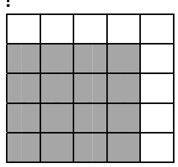
Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. If a dance floor has an area of $256\ \mathrm{m}^2$, could the dance floor be a square?
2. Determine what number and its square root can be represented by this grid if the whole square represents 1?



3. Find the square root of 289 using a “guess and test” strategy and prime factorization.
4. Are $30, 1.6\ and\ \frac{2}{5}$ perfect squares? Explain your reasoning for each.
5. Identify the error made in each case.

   $a)\ \sqrt{16} = 8$ $\qquad$ $b)\ \sqrt{0.036} = 0.6$

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N6: Determine an approximate square root of positive rational numbers that are non-perfect squares.** **[C, CN, PS, R, T]**			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
N1 Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). **N2** Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).	**N6** Determine an approximate square root of positive rational numbers that are non-perfect squares.	**AN1** ***Number, Relations and Functions 10*** Demonstrate an understanding of factors of whole numbers by determining the prime factors; greatest common factor; least common multiple; square root; cube root.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students approximated the square root of non-perfect squares, up to $\sqrt{144}$ in Grade 8. They used the perfect square benchmarks to enable them to state, between which two whole numbers the square root fell. For example, $\sqrt{27}$ lies between 5 and 6. They then were able to state that the square root was closer to 5 because the $\sqrt{27}$ is closer to $\sqrt{25}$ than to the $\sqrt{36}$. Reference may have been made to the fact that the square root of non-perfect squares always result in non-terminating, non-repeating decimals which are irrational numbers, numbers that cannot be expressed in the form $\frac{a}{b}$. Calculators may have been used to see the decimal approximations which remain an approximation no matter how many decimal places are retained in an irrational number.

In Grade 9, students will be required to estimate the square root of rational numbers in fraction and decimal form. Again they will be using benchmark perfect squares to help with their estimates using various strategies.

For example, $\sqrt{0.79} \approx \sqrt{0.81} = 0.9$ so $\sqrt{0.79} \approx 0.9$ and students should understand that the answer is a little less than 0.9.

Fractions can be addressed in a similar manner and in a couple of different ways. For example:

Method 1: $\sqrt{\frac{8}{15}} \approx \sqrt{\frac{9}{16}} = \frac{3}{4}$, so $\sqrt{\frac{8}{15}} \approx \frac{3}{4}$

Method 2: $\frac{8}{15}$ is a little more than $\frac{1}{2}$ which equals 0.5. $\sqrt{0.5} \approx \sqrt{0.49} \approx 0.7$, so $\sqrt{\frac{8}{15}} \approx 0.7$

Note: $\approx$ and $\doteq$ and $\cong$ are all used to symbolize “approximately equal to”.

SCO: **N6: Determine an approximate square root of positive rational numbers that are non-perfect squares.**
[C, CN, PS, R, T]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Estimate the square root of a given rational number that is not a perfect square using the roots of perfect squares as benchmarks.
- Determine an approximate square root of a given rational number that is not a perfect square using technology, e.g., calculator, computer.
- Explain why the square root of a given rational number as shown on a calculator may be an approximation.
- Identify a number with a square root that is between two given numbers.

SCO: **N6: Determine an approximate square root of positive rational numbers that are non-perfect squares.**
[C, CN, PS, R, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

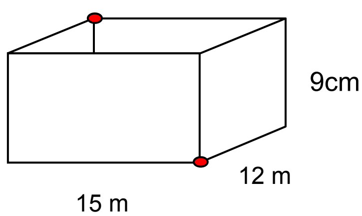
- Use perfect squares as benchmarks and have students estimate $\sqrt{0.24}$.
- Ensure that students are comfortable with the perfect square benchmarks from 1 to 400. The pattern in the difference between perfect squares can be explored to help students go beyond 144 to determine which numbers are perfect squares. Compare the differences of two perfect squares and investigate the patterns.

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, ___, ___, _____

+3 +5 +7 +9 etc

Suggested Activities

1. A spider has taken up residence in a small cardboard box that has a length of 15 cm, a width of 12 cm and a height of 9 cm. What is the length, in centimetres, of a spider web that will carry the spider in a straight line from the upper left back corner of the box to the lower right front corner of the box (from ● to ●)?



2. Given the area of a circle, use the following formula to find the radius of that circle:

$$r = \sqrt{\frac{A}{x}}$$

3. Place the square roots of the non-perfect squares, $\sqrt{8}$, $\sqrt{2}$ and $\sqrt{6}$, on a number line using benchmarks of perfect squares.

Possible Models: calculators, hundredths grids, number lines

SCO: **N6: Determine an approximate square root of positive rational numbers that are non-perfect squares.**
[C, CN, PS, R, T]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. Estimate the following square roots using benchmarks. Explain your strategy.

   $\sqrt{300}$ $\sqrt{0.45}$ $\sqrt{\frac{24}{65}}$

2. Find the approximate square root of 1.4 using the guess and check strategy and then use your calculator to verify. Show each value you tried and the result of squaring each value.
3. Using the √ symbol on his calculator, Paul found the square root of 87 to be exactly 9.327379053. Is he correct? Explain why or why not.
4. Identify:
   a) a whole number with a square root that lies between 6 and 7;
   b) a rational number with a square root that lies between 0.7 and 0.8.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **PR1: Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.**
[C, CN, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
PR1 Graph and analyze two-variable linear relations.	**PR1** Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.	**RF4** ***Number, Relations and Functions 10*** Describe and represent linear relations, using words, ordered pairs, tables of value, graphs, equations.

ELABORATION

Guiding Questions:
• What do I want my students to learn?
• What do I want my students to understand and be able to do?

Students have been exposed to patterns through interpretation of graphs of linear relations in earlier grades.

From a pictorial pattern students should be able to identify and write the pattern rule and create a table of values in order to write an expression to represent the situation. When an oral or written pattern is given, students should be able to write an expression directly from that pattern.

Linear expressions have both a variable and a constant, either of which can be 0. This connection is seen in situations involving membership fees, where there is an initial fee (constant value) and a usage fee (variable value). It is important to make a clear distinction between the two. It is also necessary to describe a context represented by a linear equation.

When students are looking at the table of values below, they should look at the pattern and recognize a constant change between the values (an increase of 6 between the term values).

Term Number (n)	1	2	3	4	5
Term (t)	2	8	14	20	26

Students should recognize that multiplying the term number, n, by 6 always results in four more than the associated term, t. Therefore, the term value (t) can be determined by subtracting 4 from 6n. As an equation, the pattern is represented by t = 6n – 4. Students should verify their equation by substituting values from the table (when n=5, t=26). Students should use their equation to solve for any value of n or t.

SCO**: PR1: Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.**
[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Write an expression representing a given pictorial, oral or written pattern.
- Write a linear equation to represent a given context.
- Describe a context for a given linear equation.
- Solve, using a linear equation, a given problem that involves pictorial, oral and written linear patterns.
- Write a linear equation representing the pattern in a given table of values and verify the equation by substituting values from the table.

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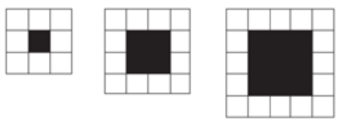
PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:
- Provide students with the opportunity to explore various patterns by explaining each pattern using words and writing an equation to represent a situation. For example, the relationship between the number of bricks (b) around a square fire pit with side lengths (s) is represented by the equation $4s + 4 = b$.

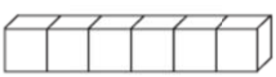


- Provide students with experiences to develop the ability to write equations for situations which are described in words. For example: *Ralph rents snowboards for $10.50 per hour, but requires a $25 non-refundable deposit. How much will it cost to rent Ralph's snowboard and use it for 2 hours? 3 hours? 6 hours? 10 hours?*
- From looking at the patterns that develop, or from the wording of the problem, students should be able to calculate the total cost as equal to $\$25$ deposit + $\$10.50$ per hour, written in equation form as $c = 25 + 10.50h$. This can then be used to calculate cost depending on the number of hours rented.
- Develop equations from patterns expressed in table form. For example, given the following table of values, have students write an equation and then verify it by substituting values from the table.

Term Number (n)	1	2	3	4	5
Term (t)	39	35	31	27	23

Suggested Activities

1. To explore patterns, have students use linking cubes to determine the number visible faces found on from 1 to 6 (or more) cubes linked together. Do not include the bottom which is not visible.



Number of cubes (c)	1	2	3	4	5	6
# visible faces (f)	5	8	11	14	17	20

To develop an understanding of patterns with relation to linear equations have students:
- Describe any patterns you see in the table and describe how the patterns are visible in the cubes.
- After the first few cubes try to predict the value for 5 cubes, for 6 cubes.
- Complete: "If you tell me how many cars are in the train, I can tell you the number of faces by ..."
- Exchange your written statements with another student and then use the statement written to predict the number of faces of a train with 100 cars.
- Describe and explain your patterns in words, if there are 'c' cars in the train.
- Change the words into mathematical symbols, describing the pattern in an algebraic sentence. Explain the meaning of each of the coefficients in the equation i.e. $f = 3c + 2$, explain the meaning of '3' and '2'.

Possible Models: linking cubes, graph paper

SCO**: PR1: Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.**
[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. Tom is getting in shape. The first day he does 9 sit-ups, the second day 13, the third day 17, the fourth day 21. Write an equation to represent this situation. If he continues in this way, how many sit-ups will he do on the 5th day? 6th day? 10th day? 20th day? 50th day? 60th day? What restrictions come into play as this pattern continues?

2. Write a linear equation to represent the pattern in the given table of values. Describe a context for the equation.

x	y
1	10.50
2	11.00
3	11.50
4	12.00

3. Given the equation, $y = 2x + 5$, describe this relation in words. Make up a problem which could be solved using this equation.

4. Your class is planning a trip to the zoo. The school will have to pay $200 for the bus plus $5 per student. How much will it cost for 42 students?

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **PR2: Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.** **[C, CN, PS, R, T, V]**			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
PR1 Graph and analyze two-variable linear relations.	**PR2** Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.	**RF1** ***Number, Relations and Functions 10*** Interpret and explain the relationships among data, graphs and situations. **RF 5** ***Number, Relations and Functions 10*** Develop and use the distance formula and midpoint formula. **RF6** ***Number, Relations and Functions 10*** Determine the characteristics of the graphs of linear relations, including the intercepts; slope; domain; range.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students will be asked to describe patterns from graphs. They will be expected to use terminology such as increase and decrease to describe the relationship between the two variables. Students have had experience with this concept in grades 7 and 8, and this is an extension of patterns to include vertical and horizontal lines. Vertical and horizontal lines can be represented by linear equations that involve only one variable. This concept may be difficult for students to grasp at first, therefore multiple opportunities to explore this concept must be provided. In this case, students will realize that as one variable changes the other stays constant. This will be an indication that the graph will be a horizontal or vertical line.

Situations may include either discrete or **continuous data**. Discrete data can only have a finite or limited number of possible values. Generally discrete data are counts: number of students in class, number of tickets sold, how many Christmas trees were purchased. A graph of discrete data has plotted points, but they are not joined together. **Continuous data** can have an infinite number of possible values within a selected range, as seen in measurements of temperature and time.

Students will be asked to **interpolate** and **extrapolate** graphs in order to solve problems. Interpolation consists of estimating a value between two given values; while extrapolation consists of estimating a value beyond a given set of values. In order to extrapolate, students must extend the pattern beyond the given data. When students are interpolating and extrapolating with discrete data, the points are not to be joined when extending the pattern.

The intent of this outcome is to explore the patterns and represent them by linear equations with the use of graphs and tables only. This will create the foundation for rate of change (slope) and the slope y-intercept form of a linear equation ($y = mx + b$) that will be explored in later grades.

SCO: **PR2: Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.**
[C, CN, PS, R, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Describe the pattern found in a given graph.
- Graph a given linear relation, including horizontal and vertical lines.
- Match given equations of linear relations with their corresponding graphs.
- Extend a given graph (extrapolate) to determine the value of an unknown element.
- Interpolate the approximate value of one variable on a given graph given the value of the other variable.
- Extrapolate the approximate value of one variable from a given graph given the value of the other variable.
- Solve a given problem by graphing a linear relation and analyzing the graph.

SCO: **PR2: Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.**
[C, CN, PS, R, T, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with a variety of problems in which they will graph a linear relation and use interpolation and extrapolation to solve the problem.
- Provide students with graphs of discrete data arranged horizontally and vertically. Students should create a table of values from the graph, write an equation by recognizing the pattern in the data and be able to describe a situation to represent each graph.
- Provide students with various graphs and linear relations and ask them to match the graph with the equation. Students could also be asked to describe the pattern within the graphs.

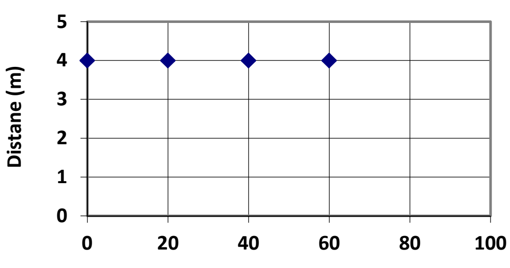
Suggested Activities

1. A taxi cab charges the rates shown in the following table.

Length of trip (km)	5	10	15
Total cost ($)	9.25	15.50	21.75

a) Plot these points on a co-ordinate system.
b) Discuss if these points should be joined.
c) Determine the equation.
d) Explain why the graph does not start at the origin.
e) From the graph, find the length of a trip which costs $25.
f) From the graph, find the cost of a trip of 12 km.

2. Give students the following graph and have them complete the activities below.



a) Create a table of values.
b) Describe the pattern found in the graph.
c) Describe a situation that the graph might represent.
d) Write a linear equation.

Possible Models: graph paper, graphing calculator

SCO: **PR2: Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.**
[C, CN, PS, R, T, V]

ASSESSMENT STRATEGIES

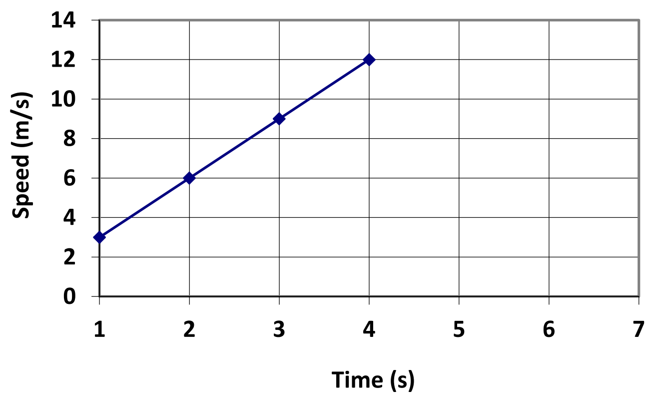
Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. Create a table of values and sketch a graph for the linear equation: $y = 7x - 4$.
2. You have just purchased a new cell phone. The phone plan costs $10 per month and $0.10 per text message. Create a graph to represent the situation. Estimate the cost of sending 100 text messages using the graph.
3. Given the following graph describe the pattern and write the equation. Describe a situation that could result in the graph.



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **PR3: Model and solve problems using linear equations of the form:**

$$\boldsymbol{ax = b;\quad \frac{x}{a} = b, a \neq 0;\quad ax + b = c;\quad \frac{x}{a} + b = c, a \neq 0;\quad ax = b + cx;}$$

$$\boldsymbol{a(x+b) = c;\quad ax + b = cx + d;\quad a(bx+c) = d(ex+f);\quad \frac{a}{x} = b,\quad x \neq 0}$$

$\boldsymbol{where\ a, b, c, d, e\ and\ f\ are\ rational\ numbers.}$

[C, CN, PS, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
PR2 Model and solve problems using linear equations of the form: • *ax = b;* • $\frac{x}{a} = b$ *, a ≠ 0;* • *ax + b = c;* • $\frac{x}{a}$ *+ b = c a ≠ 0;* • *a(x + b) = c* concretely, pictorially and symbolically, where *a*, *b* and *c* are integers.	**PR3** Model and solve problems using linear equations of the form: $ax = b;\ \frac{x}{a} = b, a \neq 0;$ $ax + b = c;\ \frac{x}{a} + b = c, a \neq 0;$ $ax = b + cx;\ a(x+b) = c;$ $ax + b = cx + d;$ $a(bx+c) = d(ex+f);$ $\frac{a}{x} = b,\quad x \neq 0$ $where\ a, b, c, d, e\ and\ f$ $are\ rational\ numbers.$	**A1** ***Geometry, Measurement and Finance 10*** Solve problems that require the manipulation and application of formulas related to perimeter, area, the Pythagorean theorem, primary trigonometric ratios, income **RF10** ***Number, Relations and Functions 10*** Solve problems that involve systems of linear equations in two variables, graphically and algebraically. .

ELABORATION

Guiding Questions:
• *What do I want my students to learn?*
• *What do I want my students to understand and be able to do?*

In grade 8, students had experience solving one and two-step equations in the form of
$ax = b;\quad \frac{x}{a} = b, a \neq 0;\quad ax + b = c;\quad \frac{x}{a} + b = c, a \neq 0;\quad a(x+b) = c.$
A review of the various informal methods to solve equations developed in grade 7 and 8 may be necessary. These could include the use of algebra tiles, inspection, systematic trials (guess and test).

In grade 9, students will continue to solve equations which include integers and rational numbers, when the “unknown” is found on both sides of the equal sign or found in the denominator, and when more than two steps are required to solve the equation.

In problem solving situations, students should be aware that once they acquire a solution, it can be checked for accuracy by substitution into the original equation.

Proper use of vocabulary should be modelled: relationship, equality, algebraic equation, distributive property, like terms, “balancing”, the zero principle, the elimination process, isolating variables, coefficient, constant, equation versus expression.

Models can be used to help students develop understanding of the process of solving equations.

SCO: **PR3: Model and solve problems using linear equations of the form:**

$$ax = b; \quad \frac{x}{a} = b, a \neq 0; \quad ax + b = c; \quad \frac{x}{a} + b = c, a \neq 0; \quad ax = b + cx;$$

$$a(x + b) = c; \quad ax + b = cx + d; \quad a(bx + c) = d(ex + f); \quad \frac{a}{x} = b, \quad x \neq 0$$

$$where\ a, b, c, d, e\ and\ f\ are\ rational\ numbers.$$

[C, CN, PS, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Model the solution of a given linear equation using concrete or pictorial representations, and record the process.
- Determine, by substitution, whether a given rational number is a solution to a given linear equation.
- Solve a given linear equation symbolically.
- Identify and correct an error in a given incorrect solution of a linear equation.
- Represent a given problem using a linear equation.
- Solve a given problem using a linear equation and record the process.

SCO: **PR3: Model and solve problems using linear equations of the form:**

$$ax = b; \quad \frac{x}{a} = b, a \neq 0; \quad ax + b = c; \quad \frac{x}{a} + b = c, a \neq 0; \quad ax = b + cx;$$
$$a(x + b) = c; \quad ax + b = cx + d; \quad a(bx + c) = d(ex + f); \quad \frac{a}{x} = b, \quad x \neq 0$$
$$\textit{where } a, b, c, d, e \textit{ and } f \textit{ are rational numbers.}$$

[C, CN, PS, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use diagrams and concrete materials to help students understand the steps needed to isolate the variable.
- Model the solving of equations with variables on both sides, using balancing strategies of balance scales (when terms are positive) and algebra tiles (when terms are negative). Examples are shown in the *Whole Class/Group/Individual Assessment* section.
- Solve equations with rational numbers in fraction or decimal form (not easily modelled with balance scales or algebra tiles) by doing the same action on both sides of the equation.
- Use equations to model and then solve problems.

Suggested Activities

1. Have students develop an equation for the following situations, and use it to answer the question(s):
   a) To raise money the student council organized a dance. They hired a band and rented electronic equipment at a cost of $800. Participation and school spirit is important to the council so they charged only $5 per ticket. How many tickets would they need to sell to break even? To make a thousand dollars? To make two thousand dollars?
   b) Paul starts work 3 hours earlier than his sister Katie. Both work at the local grocery store. Katie earns $12 and Paul earns $8. Paul wants to know how many hours he will have to work in order to earn the same amount of money as his sister.

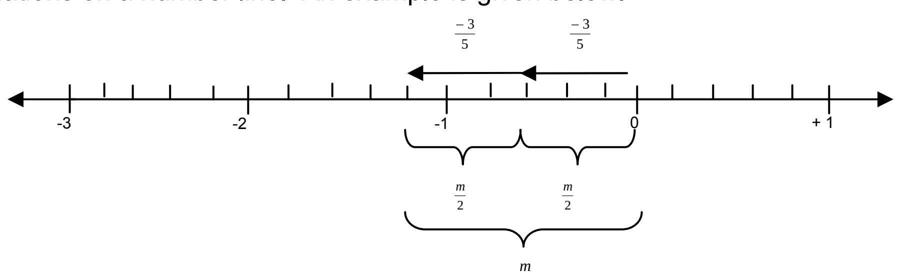
2. Have students solve the following equations using inverse operations or inspection.

   $3x = 0.6$ $\qquad \frac{m}{5} = 0.15$ $\qquad \frac{0.32}{p} = 0.08$ $\qquad 5t + 0.20 = 0.60$

3. Model simple equations on a number line. An example is given below.

$$\frac{m}{2} = -\frac{3}{5}$$

$$m = -\frac{6}{5}$$



Possible Models: algebra tiles, balance scales (pan or beam), number line, hundredths grid, coins

SCO**: PR3: Model and solve problems using linear equations of the form:**

$$\boldsymbol{ax = b;\quad \frac{x}{a} = b, a \neq 0;\quad ax + b = c;\quad \frac{x}{a} + b = c, a \neq 0;\quad ax = b + cx;}$$

$$\boldsymbol{a(x+b) = c;\quad ax + b = cx + d;\quad a(bx+c) = d(ex+f);\quad \frac{a}{x} = b,\quad x \neq 0}$$

$\boldsymbol{where\ a, b, c, d, e\ and\ f\ are\ rational\ numbers.}$

[C, CN, PS, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

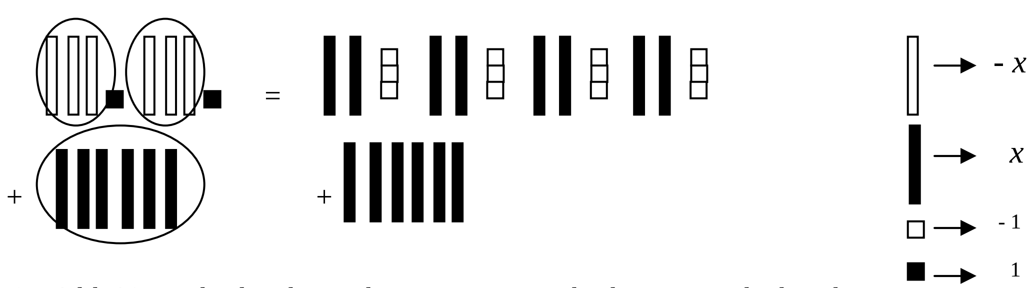
Whole Class/Group/Individual Assessment

1. Give the equation that shows that the perimeter of a rectangle is 36 m, if the length of the rectangle is 2 m less than its width. Solve for the length and width.

2. Solve the following, using tiles, and record each step algebraically.

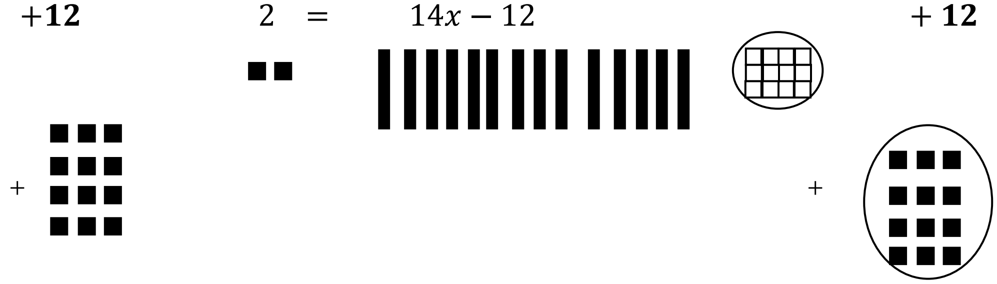
$$\begin{aligned} 2(-3x+1) &= 4(2x-3) \\ -6x+2 &= 8x-12 \end{aligned}$$

$\boldsymbol{Step\ 1}$ $Add\ \ 6\boldsymbol{x}\ \ to\ both\ sides\ and\ remove\ every\ tile\ that\ is\ matched\ with\ its\ negative,$ $(-x+x) = 0$



$\boldsymbol{Step\ 2}$ $Add\ \ \boldsymbol{12}\ \ to\ both\ sides\ and\ remove\ every\ tile\ that\ is\ matched\ with\ its\ negative$ $(+1-1) = 0$

$\boldsymbol{+12}$ $\quad 2 = 14x - 12 \quad$ $\boldsymbol{+12}$



$\boldsymbol{Step\ 3}$ $Group\ each\ x\ with\ an\ even\ number\ of\ single\ tiles\ \boldsymbol{14 = 14x}, so\ each\ x = 1$

SCO: **PR3: Model and solve problems using linear equations of the form:**

$$\boldsymbol{ax = b;\quad \frac{x}{a} = b, a \neq 0;\quad ax + b = c;\quad \frac{x}{a} + b = c, a \neq 0;\quad ax = b + cx;}$$
$$\boldsymbol{a(x + b) = c;\quad ax + b = cx + d;\quad a(bx + c) = d(ex + f);\quad \frac{a}{x} = b,\quad x \neq 0}$$
$\boldsymbol{where\ a, b, c, d, e\ and\ f\ are\ rational\ numbers.}$

[C, CN, PS, V]

3. Estimate a solution to the following equation. Justify your estimate; solve and verify.

$$\frac{x}{2} - 3 = 1\frac{1}{6}$$
.

4. Brenda and Thomas want to buy a $199 portable iPod Touch player. Brenda has $45 and saves $15 per week. Thomas has $70 and saves $12.50 per week. Who will be able to buy the iPod Touch player first? Solve using a linear equation.

5. *The Telegraph Journal* can be delivered to your house for $0.70 per copy plus a $25.00 yearly subscription fee. *The Globe and Mail* can be delivered to your house for $0.75 per copy plus a $20.00 yearly subscription fee. Determine how many copies are delivered before the cost is the same.

6. Leah solved the following equation. Check for any errors. If any were made, indicate where and make the necessary changes to correct them.

$$\frac{1}{3}(x - 2) = 5(x + 6)$$
$$3(x - 2) = 5(x + 6)$$
$$3x - 6 = 5x + 30$$
$$3x - 6 + 6 = 5x + 30 + 6$$
$$3x - 5x = 5x - 5x + 36$$
$$-2x = 36$$
$$\frac{-2x}{-2} = \frac{36}{-2}$$
$$x = -18$$

7. Solve the following:

$a)\ 2(3x - 6) = \frac{1}{2}(4x + 2)$ $\quad b)\ \frac{x}{4} = \frac{7}{10}$ $\quad c)\ \frac{1}{3}x + 8 = -1.4$

$d)\ \frac{k}{3} - \frac{1}{2} = -1\frac{3}{4}$ $\quad e)\ 8(3d - 2) = -12.32$ $\quad f)\ \frac{5}{m} + 7 = -3$

$g)\ 2.3(h - 1.7) = 4.2(h + 1.3)$ $\quad h)\ \frac{x}{3} + \frac{1}{2} = \frac{5}{6}$ $\quad i)\ \frac{1}{2}b - 5 = 4 - b$

$j)\ -2(1 - c) = -3(2 - c)$ $\quad k)\ \frac{t}{3} - \frac{3t}{4} = 10$ $\quad l)\ \frac{-5}{x} = -2.$

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **PR4: Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.**
[C, CN, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
	PR4 Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.	

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Solving linear inequalities is new at the grade 9 level and will build on their previous knowledge of linear equations. An **inequality** is defined as a mathematical sentence that compares two expressions that may or may not be equal. Students need to realize that unlike linear equations which have a single solution, inequalities will have many solutions.

Students will learn that the operation rules for inequalities are the same as for linear equations with the exception of the multiplication and division of negatives. The rationale for the reversal of the inequality sign should be developed through examples. Emphasis should be placed on the graphing of solutions on a number line when x is a member of the whole or integer system and the real number system, in order to gain a good understanding of the solution as a set of values rather than a single value.

Where possible an effort should be made to have students describe a problem or situation as an inequality to be solved and then represented on a number line. Many of these problems are real life situations. This is a good opportunity for teachers to discuss with students that there may or may not be limits on these inequalities that are created, depending on the context of the problem. For example, if discussing speed of a vehicle as less than $60\ km/h$, or $v < 60$, students should realize that speed cannot drop below zero.

SCO: **PR4: Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.**
[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Translate a given problem into a single variable linear inequality using the symbols ≥, >, < or ≤.
- Determine if a given rational number is a possible solution of a given linear inequality.
- Generalize and apply a rule for adding or subtracting a positive or negative number to determine the solution of a given inequality.
- Generalize and apply a rule for multiplying or dividing by a positive or negative number to determine the solution of a given inequality.
- Solve a given linear inequality algebraically and explain the process orally or in written form.
- Compare and explain the process for solving a given linear equation to the process for solving a given linear inequality.
- Graph the solution of a given linear inequality on a number line.
- Compare and explain the solution of a given linear equation to the solution of a given linear inequality.
- Verify multiple solutions of a given linear inequality using substitution.
- Solve a given problem involving a single variable linear inequality and graph the solution.

SCO: **PR4: Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.**
[C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

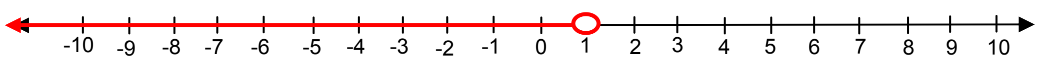
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

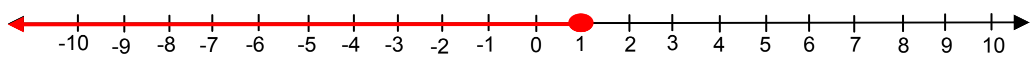
Consider the following strategies when planning lessons:

- Have students start with a statement they know is true, for example $5 > -2$. Have them explore the operations of addition, subtraction, multiplication and division of both positive and negative integers. Discuss the results. Use the outcomes of this activity to generalize rules for solving inequalities.
- Once students have an understanding of operation rules have them expand to questions involving a single variable, such as $-2x - 5 < 3$.
- Explore the difference between $<, >, \leq, \geq$ and how to represent them on the number line.

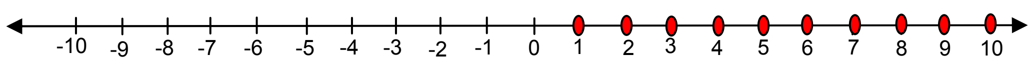
  For example $x < 1$ could look like:



  Whereas $x \leq 1$, would look like:



- Introduce questions with answers that are discrete values and graph the solutions. For example $x \geq 1, x \in \mathrm{I}\ (Integers)$



Suggested Activities

1. Determine if the values in the following table satisfy the inequality given.

Inequality	Values
$x > 3$	$5, \quad 7, \quad 9, \quad 10$
$-3x + 12 < 36$	$-9, \quad -10, \quad -15.2$
$\frac{x}{4} + 6 \geq -2$	$-10, \quad 15, \quad \frac{2}{3}, \quad 7$

2. Graph the following inequalities on a number line.

   $a)\ 3x - 2 \leq -20$ $\quad b)\ 7 - 3x \leq 22$ $\quad c)\ 2 + \frac{2}{3}x > \frac{1}{2}$ $\quad d)\ 2 - 5x > 2x + 16$

3. Glen received grades of 75%, 82%, and 78% on his first three summative assessments. They are all equally weighted. What mark is required on the next summative assessment to achieve an average of at least 80%? To solve the problem, set up an inequality that would help you solve the problem, understanding that there is a maximum mark that Glen could get in the final assessment.
4. Discuss the difference between $2x + 1 = 5$ and $2x + 1 > 5$.
5. Julie bought a $50 prepaid card for her cell phone usage. She has to pay a monthly rate of $15 and then $0.15 per text message. If she only texts, how many text messages can she send this month? How would you graph the solution?

Possible Models: number lines

SCO: **PR4: Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.**
[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

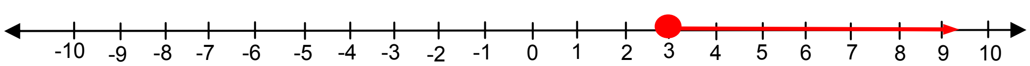
Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

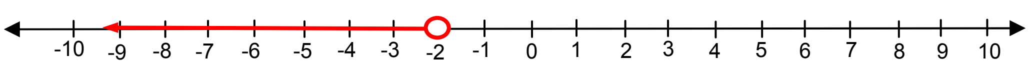
Whole Class/Group/Individual Assessment

1. Write in journals or discuss the following.
   a) Explain why $3n - 2 > 8$ and $3n + 4 < 14$ do not have any solutions in common. Modify one of the inequalities so that they can have exactly one solution in common.
   b) Create three inequalities that are equivalent and explain why they are equivalent.
   c) Create a real life problem that could be represented by the inequality, $3x + 12 < 21$.
2. Determine, using substitution, if the values, $-5, -2.3, -2.4,\ 5,\ 7.2,\ 10$ satisfy the inequality $-3x + 4 < 7$.
3. Write an inequality for the following graphs:

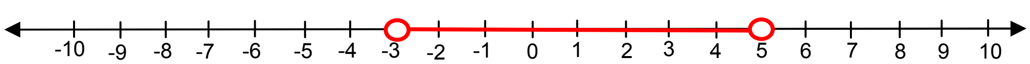
a)



b)



c)



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **PR5: Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V]**			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
	PR5 Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).	**AN4** ***Number, Relations and Functions 10*** Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically. **AN5** ***Number, Relations and Functions 10*** Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

These new terms are introduced in this section:

- The parts of an algebraic expression are called **terms**. For example 2, $3x^2$ and $4x$ are all terms.
- A **variable** term can change, and is typically represented by x, y, or z.
- A **constant** term is a number that is not linked to a variable and so does not change.
- A **co-efficient** is the number by which the variable is multiplied.
- A **polynomial** is the algebraic expression of terms connected by the operations of addition or subtraction.
- The **degree of a term** is the sum of the exponents of the variables in a single term. For example, the degree of $4x^2y$ is 3. A variable with no exponent showing is understood to have an exponent of one.
- The **degree of a polynomial** is the highest degree of any term in the polynomial.
- All expressions with one or more terms are called polynomials. Some polynomials are named by the number of terms they contain. For example, a **monomial** has one term, a **binomial** has two terms and a **trinomial** has three terms.

Students will be familiar with the use of algebra tiles for modeling linear situations that were used in Grade 8. With the introduction of second degree polynomials in Grade 9 students will need to be introduced to x^2 **tiles**.

At this stage, students should be comfortable changing from models and pictorial representations to polynomial expressions and polynomial expressions back to models and pictorial representation. Rearranging polynomial expressions to show that some expressions are equivalent should also be explored.

SCO: **PR5: Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).**
[C, CN, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Create a concrete model or a pictorial representation for a given polynomial expression.
- Write the expression for a given model of a polynomial.
- Identify the variables, degree, number of terms and coefficients, including the constant term, of a given simplified polynomial expression.
- Describe a situation for a given first degree polynomial expression.
- Match equivalent polynomial expressions given in simplified form, e.g., $4x - 3x^2 + 2$ is equivalent to $-3x^2 + 4x + 2$.

SCO: **PR5: Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).**
[C, CN, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

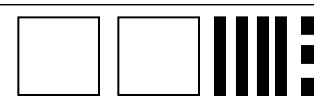
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use diagrams and concrete materials to demonstrate the idea of converting models to expressions.

Suggested Activities

1. Given a polynomial, students should identify the terms, degree, variables, coefficients and constants in each, and represent a model. Provide Students with a chart similar to below and insert any polynomial expression or model and complete the remaining cells.

Expression	# of terms	Degree	Variable(s)	Coefficient(s)	Constant	Model
3	1	0	none	none	3	
x^2+2x-3	3	2	x	$1, 2$	-3	
$4x-2x^2+3$	3	2	x	$4, -2$	3	

Possible Models: algebra tiles

SCO: **PR5: Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).**
[C, CN, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. Create a polynomial expression for each of the following descriptions. For example, a polynomial of degree 2, with a constant of -4, would be $x^2 - 4$
   a) A binomial with a coefficient of 4.
   b) A trinomial of degree 2, with coefficients of 4 and -1.
   c) A binomial with no constant term.

2. Compare the polynomial in **List A** with the corresponding polynomial in **List B,** and determine if they are equal or unequal.

List A	List B
$3x - x^2 - 2$	$-x^2 + 3x - 2$
$7 + 2x + x^2$	$x^2 - 2x + 7$
$3x - 5$	$5 - 3x$

3. Describe a real life situation that would model the binomial expression $2x + 3$.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO**: PR6: Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree 1 or 2).**
[C, CN, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
	PR6 Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2).	**AN4** ***Number, Relations and Functions 10*** Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically. **AN5** ***Number, Relations and Functions 10*** Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

ELABORATION

Guiding Questions:
• What do I want my students to learn?
• What do I want my students to understand and be able to do?

Students have been exposed to the notion of a variable through the solving of equations since Grade 5 and algebraic reasoning and the notion of equality is a focus of the new curriculum beginning at Kindergarten. As part of the continued development of algebra, students should be given a variety of ways to relate to the symbolism. One connection which may be useful to students relates to measurement situations. For example, can we add 3m (metres) + 5m^2 (square metres) and get 8 of something? Students realize that the units must be the same in order to add or subtract them. This should enable them to more easily transfer to the concept of like and unlike terms. The use of algebra tiles to model terms is crucial to strengthen this transfer, so that students can visualize the difference between x and x^2 or between x and $+3$.

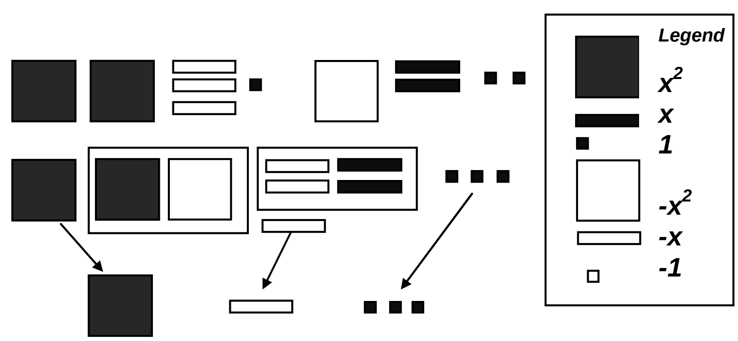
Students have also worked with integers and modeled operations with 2 colour counters so they have experience with the idea of positive and negative numbers (terms) and the zero principle.

Initially, time should be spent on modeling and identifying like and unlike terms before the addition and subtraction of polynomials is introduced. It is important that when the addition and subtraction of polynomial expressions are modeled the students record the expressions and the process symbolically. For example:

$(2x^2 - 3x + 1) + (-x^2 + 2x + 2)$
Combine like terms

$= (2x^2 - x^2) + (-3x + 2x) + (+1 + 2)$
Remove zeros

$= +x^2 + (-x) + (+3)$
Or $= x^2 - x + 3$



It is important that students move from the concrete to the pictorial to the symbolic, but initially either the concrete and symbolic or the pictorial and symbolic should be done in tandem.

SCO**: PR6: Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree 1 or 2).**
[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify like and unlike terms.
- Model addition of two given polynomial expressions concretely or pictorially and record the process symbolically.
- Model subtraction of two given polynomial expressions concretely or pictorially and record the process symbolically.
- Apply a personal strategy for addition and subtraction of given polynomial expressions, and record the process symbolically.
- Identify equivalent polynomial expressions from a given set of polynomial expressions, including pictorial and symbolic representations.
- Identify the error(s) in a given simplification of a given polynomial expression.

SCO: **PR6: Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree 1 or 2).**
[C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Ensure students have opportunity to view subtraction in a variety of ways.

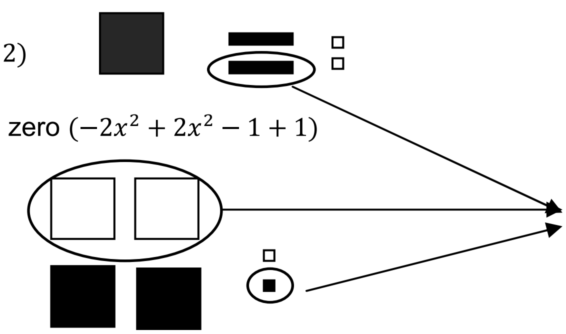
Comparison simply refers to looking at 2 quantities and the difference between them. What would make the 2 quantities equivalent? This can be done by either "adding on" to the smaller quantity or "taking away" from the larger quantity.

Taking away refers to starting with a quantity and removing or taking away a specified amount. However, to do this it may sometimes be necessary to add "zero amounts" to take away the specified amount.

For example, in the question $(x^2 + 2x - 2) - (-2x^2 + x + 1)$, you have $2x$ to start with so are able to take away one *x*. However, you begin with a $(+x^2)$ so you are not able to take away $(-2x^2)$. Likewise, you begin with (-2) so you are not able to take away $(+1)$. "Zero amounts", which do not change the value of an expression, can be added to the original expression to accommodate this situation. In this case, if you add $(-2x^2)$ and $(+2x^2)$ and also add $(+1)$ and (-1) you are then able to take away $(-2x^2 + x + 1)$.

Step 1)
$(x^2 + 2x - 2)$

Step 2) Add zero $(-2x^2 + 2x^2 - 1 + 1)$



Step 3) take away $(-2x^2 + x + 1)$

This shows how removing $(-2x^2 + 1)$ is the same as adding $(2x^2 - 1)$ which leads to the next strategy for subtraction

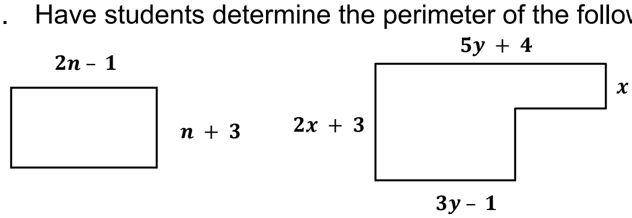
Step 4) The remaining blocks, $(3x^2 + x - 3)$, give the answer

Adding the opposite refers to approaching subtraction by first changing the question to an addition question and adding the opposite of a quantity. For example, for $(x^2 + 2x - 2) - (-2x^2 + x + 1)$ instead of subtracting the second polynomial, add the opposite values of $(+2x^2 - x - 1)$.

- Use perimeter problems as a good application for addition and subtraction of polynomials.
- Take time to show students that algebra tiles may represent different variables; not just the typical models of x^2 and x. Do not stick exclusively to x variables.

Suggested Activities

1. Have students determine the perimeter of the following shapes.



SCO: **PR6: Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree 1 or 2).**
[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

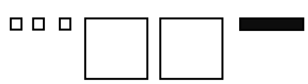
1. Identify the like terms: $5x^2,\ 3xy,\ -2x^2,\ 2x$

2. Model each sum or difference concretely (algebra tiles) or pictorially and record the steps symbolically.
   a) $(-2x^2 - 2x - 3) + (x^2 - 2x + 2)$
   b) $(x^2 + 3x - 4) - (-2x^2 + 1)$

2. Simplify using your own strategy, either concretely, pictorially or symbolically.
   a) $(2x^2 - 5x) - (-3x^2 + 2x)$
   b) $(3m^2 - 2mn - 4) + (m^2 + 2)$

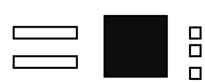
3. Identify which expressions are equivalent to $-2y^2 + y - 3$.

   $a)\ y - 3 - 2y^2$

   $b)$ 

   $c)\ y^2 - 1 + 4y - 3y^2 - 3y - 2$

   $d)\ -y^2 - 3$

   $e)$ 

4. Circle the errors in the following work.
   Step 1 $(2x^2 - 3x + 2) - (x^2 + x - 1)$
   Step 2 $2x^2 - 3x + 2 - x^2 + x - 1$
   Step 3 $x^2 - 2x - 1$

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

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Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
	PR7: Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree 1 or 2) by monomials, concretely, pictorially and symbolically.	**AN4** ***Number, Relations and Functions 10*** Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically. **AN5** ***Number, Relations and Functions 10*** Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

ELABORATION

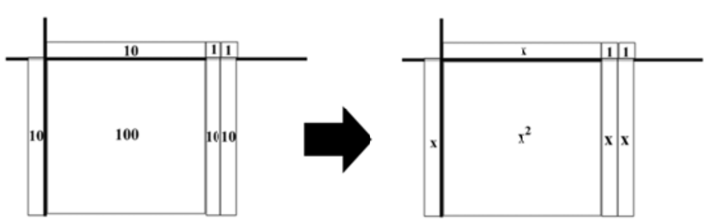
Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

This outcome, dealing with the multiplication and division of polynomial expressions, is restricted to expressions of no greater than 2^{nd} degree, to allow for the effective modeling of both operations. In other words, no part of any equation can have an exponent greater than two. The focus is on developing a deep understanding of the operations.

Students should first explore multiplication and division of a monomial by a monomial, extend to multiplication and division of a polynomial by a scalar, and then move to multiplication and division of a polynomial by a monomial.

The area model is a powerful model that students have used in numerical situations such as modeling 1-digit and 2-digit whole number multiplication using base ten materials. This modelling should transfer smoothly to the use of algebra tiles.

For example, the area model can be used to show that $10 \times (10 + 2) = 100 + 10 + 10 = 120$. Using variables, this corresponds to $x(x + 2) = x^2 + x + x = x^2 + 2x$. While students have had experience with modeling multiplication in this way, their experience with division is limited. The process to model division begins with building a rectangle with the dividend, using the divisor as one of its dimensions. For example, for $120 \div 10$, make a rectangle with one 100 square and two 10's (120). One dimension of the rectangle will be 10 and the other dimension will be one 10 and two 1's or 12 which is the solution. Likewise, for $(x^2 + 2x) \div x$, a rectangle can be built with one x^2 tile, and two x tiles. One dimension of the rectangle will be x and the other dimension will be $x + 2$ which is the solution.



SCO: **PR7: Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree 1 or 2) by monomials, concretely, pictorially and symbolically.**
[C, CN, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Model multiplication of a given polynomial expression by a given monomial concretely or pictorially and record the process symbolically.
- Model division of a given polynomial expression by a given monomial concretely or pictorially and record the process symbolically.
- Apply a personal strategy for multiplication and division of a given polynomial expression by a given monomial.
- Provide examples of equivalent polynomial expressions.
- Identify the error(s) in a given simplification of a given polynomial expression.

SCO: **PR7: Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree 1 or 2) by monomials, concretely, pictorially and symbolically.**
[C, CN, R, V]

PLANNING FOR INSTRUCTION

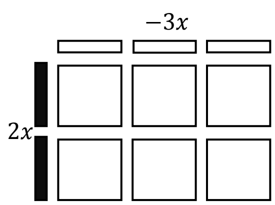
Before introducing new material, consider ways to assess and build on students' knowledge and skills.

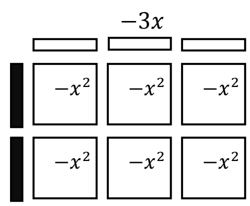
Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:
- Begin with multiplication and division of monomials using the area model and moving toward solving algebraically. For example:

For $(-3x)(2x)$, start by building a frame with the given dimensions. Fill in area with appropriate pieces. The area of $-6x^2$ is the product.





For $\frac{-6x^2}{-3x}$, start by building a rectangle (dividend) using the divisor as one of its dimensions. The other dimension, is the quotient.

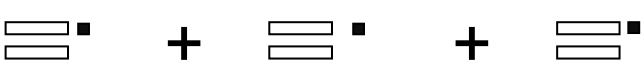
- Provide students with opportunities to draw these models using lengths of X and 1.

$(2x)(3x+2) =$

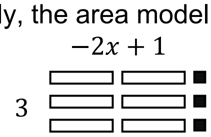
- Move toward multiplication of a polynomial by a constant and then multiplication of a polynomial by a monomial.

Suggested Activities

1. Have students model $3(-2x+1)$
   .
   Repeated addition could be used.



Alternatively, the area model could be used.



Possible Models: algebra tiles

SCO: **PR7: Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree 1 or 2) by monomials, concretely, pictorially and symbolically. [C, CN, R, V]**

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

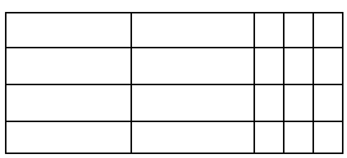
1. Demonstrate the product or quotient for each of the following using algebra tiles or diagrams and record the process symbolically.

   a) $3(2x-1)$ b) $\frac{3x^2-6x}{-3x}$

2. Find the product or quotient using a strategy of your choice.

   a) $2(x^2+3)$ b) $\frac{2x^2+8x-6}{2}$

3. Write the dimensions and area for the rectangle shown below. Write all the related multiplication and division equations.



4. Write equivalent expressions for the following:

   a) $4(2x^2+6x)$ b) $\frac{3x-9}{6}$

5. Find the missing terms in following polynomials.

   a) $3x(\square+4)=6x^2+\square$ b) $(2x^2-4x)\div\square=\square$

6. Circle the errors and correct the solution.

   a) $-4m(-2+m)=-8m+4$ b) $\frac{-12y+6}{6}=-2y$

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS1: Solve problems and justify the solution strategy using circle properties including:**
- **the perpendicular from the centre of a circle to a chord bisects the chord**
- **the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc**
- **the inscribed angles subtended by the same arc are congruent**
- **a tangent to a circle is perpendicular to the radius at the point of tangency.**

[C, CN, PS, R, T, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
	SS1 Solve problems and justify the solution strategy using circle properties including: • the perpendicular from the centre of a circle to a chord bisects the chord • the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc • the inscribed angles subtended by the same arc are congruent • a tangent to a circle is perpendicular to the radius at the point of tangency.	

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students have explored circles in grade 7 in the form of radius, diameter, circumference and area. They have developed formulas for these topics through exploration.

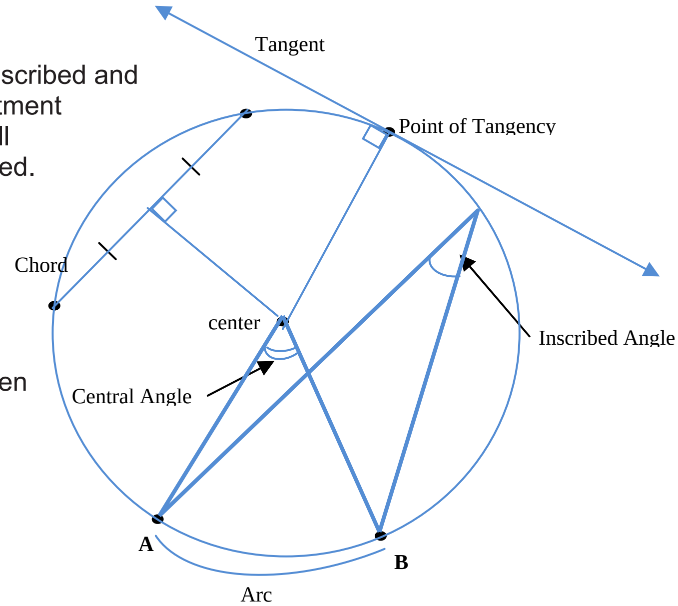
In grade 9, students will need to develop an understanding of terms relating to circle properties. A **circle** is a set of points in a plane that are all the same distance (radius) from a fixed point called the centre. A **chord** is a line segment with both endpoints on the circle. A **central angle** is an angle formed by two radii of a circle. An **inscribed angle** is an angle formed by two chords that share a common endpoint. An **arc** is a portion of the circumference of the circle. A **tangent** is a line that touches the circle at exactly one point, which is called the **point of tangency**.

Students will be exploring circle properties around chords, inscribed and central angle relationships, and tangents to circles. The treatment of these circle topics is not intended to be exhaustive, but will be determined to a significant extent by the contexts examined.

For chords, the perpendicular bisector will meet the centre of the circle, and conversely if a line runs from the centre and meets the chord at 90° it will bisect the chord.

The central angle of a circle will always measure twice that of any inscribed angle subtended by the same arc. And it then follows that all inscribed angles subtended by the same arc will have the same measure.

Tangents will be perpendicular to the radius of the circle at the point of tangency.



SCO: **SS1: Solve problems and justify the solution strategy using circle properties including:**
- **the perpendicular from the centre of a circle to a chord bisects the chord**
- **the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc**
- **the inscribed angles subtended by the same arc are congruent**
- **a tangent to a circle is perpendicular to the radius at the point of tangency.**

[C, CN, PS, R, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Provide an example that illustrates:
  - the perpendicular from the centre of a circle to a chord bisects the chord
  - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
  - the inscribed angles subtended by the same arc are congruent
  - a tangent to a circle is perpendicular to the radius at the point of tangency.
- Solve a given problem involving application of one or more of the circle properties.
- Determine the measure of a given angle inscribed in a semicircle using the circle properties.
- Explain the relationships among the centre of a circle, a chord and the perpendicular bisector of the chord.

SCO: **SS1: Solve problems and justify the solution strategy using circle properties including:**
- **the perpendicular from the centre of a circle to a chord bisects the chord**
- **the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc**
- **the inscribed angles subtended by the same arc are congruent**
- **a tangent to a circle is perpendicular to the radius at the point of tangency.**

[C, CN, PS, R, T, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

• What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?

• What teaching strategies and resources should I use?

• How will I meet the diverse learning needs of my students?

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

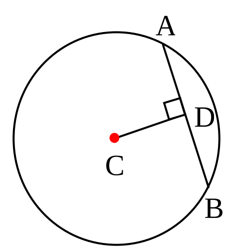
- Provide students with a handout of circles with labelled centres to explore the circle properties.
  - Ask students to draw two non-parallel chords in the same circle. Using the triangle from the geometry set have them draw a line perpendicular to each chord passing through the centre, and then measure each part of the divided chords. This exploration should lead to establishing that the perpendicular bisector of a chord will pass through the centre of the circle and conversely that the line from the centre of the circle that meets the chord at a right angle, will also bisect the chord.
  - Provide opportunities for students to draw and measure central and inscribed angles subtended by the same arc and draw conclusions from their answers.
  - Ask students to place a point outside of one of the circles and ask them to draw the two possible tangents to the circle. From the point where each tangent touches the circle (point of tangency), ask students to draw a line to the centre of the circle. Students should then measure the angle formed by the tangent and the radius. What do the students notice about these measurements?
  - Ask students to draw a diameter on one of the circles. They should then draw and measure an inscribed angle subtended by the semi circle.

Suggested Activities

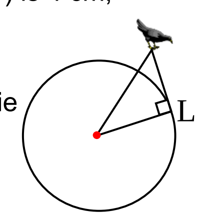
1. Challenge students with the following problem: A surveillance camera is taping people coming through the entrance of the school. While reviewing the tape, school administrators realized that the camera was broken. When shopping for a new one, the cameras available have field of view of 40° compared to the broken one which had a field of view of 80°. Where should they position the new camera to cover the same area?

2. Provide students with an arc and ask them to find the radius of the circle from which the arc was taken (could be extended to a variety of arcs).

3. Have students try the following problems:
   a) The radius of the circle to the right measures 6 cm. If the distance between the centre and the chord (CD) is 4 cm, what is the length of the chord AB?



   b) The radius of the earth is 6400 km. If a bird is 1500 m from the ground, how far is it from Leslie standing at point L?



SCO: **SS1: Solve problems and justify the solution strategy using circle properties including:**
- **the perpendicular from the centre of a circle to a chord bisects the chord**
- **the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc**
- **the inscribed angles subtended by the same arc are congruent**
- **a tangent to a circle is perpendicular to the radius at the point of tangency.**

[C, CN, PS, R, T, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

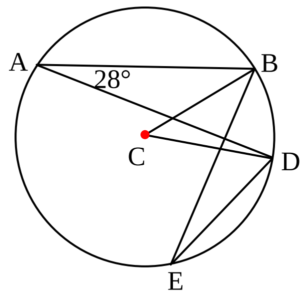
Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

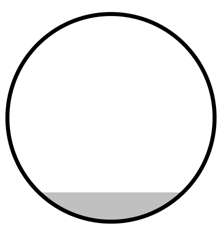
Whole Class/Group/Individual Assessment

1. You have just purchased a new umbrella to put in the centre of your wooden circular picnic table. You want to place the umbrella in the centre of the table, but the hole is not cut. Explain how you would figure out where to cut the hole for your new umbrella.

2. Find $\angle$ BCD and $\angle$ BED.



3. The diagram represents the water level in a pipe. The surface of the water from one side of the pipe to the other measures 30 mm and the inner diameter of the pipe 44 mm. What is the depth of the water?



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS2: Determine the surface area of composite 3-D objects to solve problems. [C, CN, PS, R, V]**

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
SS3 Determine the surface area of: right rectangular prisms; right triangular prisms; right cylinders to solve problems. **SS5** Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms.	**SS2** Determine the surface area of composite 3-D objects to solve problems.	**M1** ***Geometry, Measurement and Finance 10*** Demonstrate an understanding of the Système International (SI) by describing the relationships of the units for length, area, volume, capacity, mass and temperature. **M2** ***Geometry, Measurement and Finance 10*** Demonstrate an understanding of the imperial system by: describing the relationships of the units for length, area, volume, capacity, mass and temperature. **M5** ***Geometry, Measurement and Finance 10*** Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres.

ELABORATION

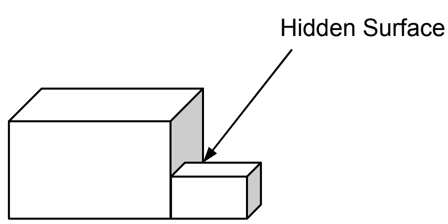
Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

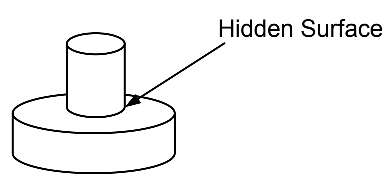
To determine the surface area of a 3D object students will sum the area of all the faces of the object. In grade 8, students have had experience calculating surface areas of right rectangular prisms, right triangular prisms, and right cylinders. In grade 9, this is extended to composite objects which combine these objects. Some students may require a review of strategies for determining area of 2-D shapes. They may also need to be reminded that area and surface area are measured in square units.

To calculate the surface area of a 3D object, students should be provided with actual nets of various 3D objects. With some experience they will no longer need the net, but will learn to visualize it, and the relationship between a 2-D net and its 3-D object.

When solids are combined into composite 3-D objects, students can use concrete items to determine what surface is hidden. Students can be guided towards realizing that the area of the overlap of surfaces needs to be subtracted from the original surface area of *both* of the original objects. As the objects are all right prisms or cylinders, the hidden surfaces will be identical to the other end that is exposed.

The students should be challenged with a variety of real world applications to find the surface area of composite objects such as buildings, containers, packaging, furniture, etc. Among other things they will need to consider that depending upon the context, different surfaces might be included. For example, when icing a tiered cake the bottom would not be included, but when wrapping a present the bottom surface would be included.





SCO: **SS2: Determine the surface area of composite 3-D objects to solve problems. [C, CN, PS, R, V]**

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Determine the area of overlap in a given concrete, composite 3-D object, and explain its effect on determining the surface area (limited to right cylinders, right rectangular prisms and right triangular prisms).
- Determine the surface area of a given concrete composite 3-D object (limited to right cylinders, right rectangular prisms and right triangular prisms).
- Solve a given problem involving surface area.

SCO: **SS2: Determine the surface area of composite 3-D objects to solve problems. [C, CN, PS, R, V]**

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

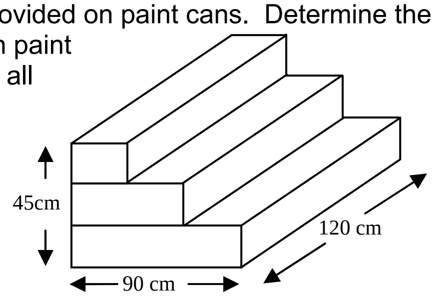
Consider the following strategies when planning lessons:

- Begin by having students sum surface areas that have already been calculated, to establish an understanding of the meaning of surface area before focussing on calculations.
- Use concrete materials to promote the understanding of which surfaces overlap and which are exposed when objects are combined into composite 3-D objects.
- Explore objects that may not appear to be composite (for example a milk carton). Students may require assistance in separating objects into familiar pieces.
- Encourage the use of estimation skills in calculations and conversion of units.
- Have students brainstorm situations that necessitate determining surface area as well as ones that require adding or removing some area calculation such as icing a wedding cake, building a garage over a permanent floor, wrapping a birthday gift or package to mail.
- Have students find the error(s) in a surface area calculation.
- As an extension, have students find one dimension given the other dimensions as well as the surface area.

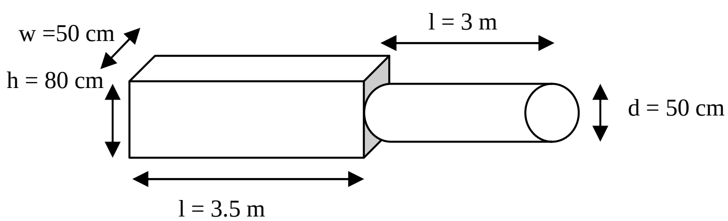
Suggested Activities

1. Build two different objects using 12 linking cubes. Determine the surface area of each object. How can symmetry help determine the surface area more efficiently. Slide the two objects together and determine which surfaces overlap. How does the overlap affects the total surface area of the new composite figure.

2. Estimated surface area coverage is provided on paint cans. Determine the surface area of a set of concrete steps to determine how much paint will be needed to cover them. Assume all steps are the same depth and height.



3. A play structure has been designed by combining a prism with a cylinder. How much fabric would be required to cover the entire surface?



Possible Models: linking cubes, geometric solids, grid paper, boxes of various shapes and sizes, paper towel rolls, Polydrons®

ASSESSMENT STRATEGIES

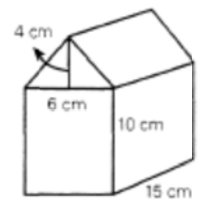
Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

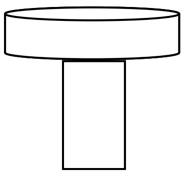
Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. To the right is a drawing of a barn made entirely of steel, scaled to 1 cm = 0.5 m. Calculate the amount of steel needed to cover the entire structure.

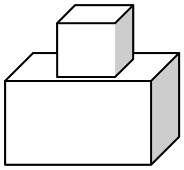


2. A coffee table has been built out of two cylinders. Calculate the 2x the surface area of the table to determine the amount needed for two coats of paint. The diameter of the large cylinder is 40 cm and the diameter of the small cylinder is 15 cm. The height of the large cylinder is 10 cm and the height of the small cylinder is 50 cm.



Note to teacher: This activity provides an excellent opportunity to discuss strategies to find surface area e.g. add up all the parts or use the fact that the part which needs to be removed “twice” could be compensated by the base.)

3. The surface area of this composite figure was calculated incorrectly to be 582 cm^2. The figure on the top is a cube with sides of 5 cm. The rectangular prism on the bottom has length of 12 cm, a width of 6 cm and a height of 8 cm. What is the correct surface area? Explain what mistake might have been made.



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS3: Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons**
[C, CN, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
	SS3 Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons.	**G4** ***Geometry, Measurement and Finance 10*** Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, solving problems

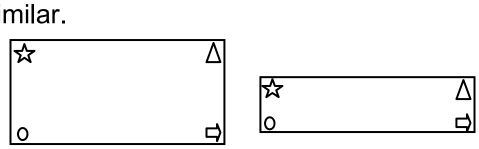
ELABORATION

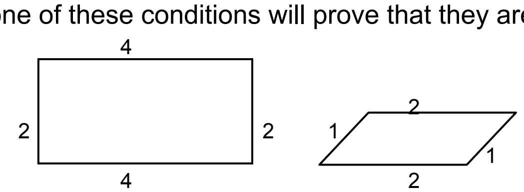
Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

This is an introduction to the concept of similarity of convex polygons. As this concept is introduced it is important make the connection between proportional reasoning and the geometric concept of **similarity**. Similar figures provide a visual representation of proportions, and proportional thinking enhances the understanding of similarity.

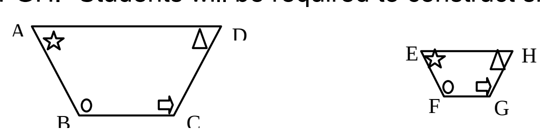
Corresponding sides are sides that have the same relative position in two geometric figures. When polygons are similar, corresponding angles are congruent and corresponding side lengths are all enlarged or reduced by the same factor (ratio).

Students should be made aware that there are two criteria that must be met for two polygons to be similar: 1) all the corresponding angles must be equal and 2) all the corresponding sides must be proportional. One could use the following examples to demonstrate the importance of satisfying both conditions for polygons. Below, the two rectangles on the left, have equal corresponding angles without being similar. Similarly, the rectangle and the parallelogram on the right, have proportional corresponding sides without being similar. Note: For triangles either one of these conditions will prove that they are similar.





In the polygons below, all angles are equal and all corresponding side lengths have been reduced by the same ratio of 1:2.The symbol ~ is used to identify similarity. For example, ABCD ~ EFGH is read, "Trapezoid ABCD *is similar to* Trapezoid EFGH." Students will be required to construct similar polygons and explain why they are similar.



Students should be exposed to a variety of situations, including pairs of similar figures that vary in orientation. The properties of similar polygons can be used to find the measures of missing sides and angles. This topic lends itself well to real life situations such as finding heights of buildings or distances which are normally difficult to measure directly, or finding the distance across a pond. Please note this outcome should be taught in conjunction with outcome SS4.

SCO: **SS3: Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons**
[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Determine, using angle measurements, and ratios of side lengths, if two or more regular or irregular polygons are similar.
- Explain why two given polygons are not similar.
- Explain the relationship between the corresponding sides of two similar polygons.
- Draw a polygon that is similar to a given polygon.
- Explain why two or more right triangles with a shared acute angle are similar.
- Solve a contextual problem that involves similarity of polygons.

SCO: **SS3: Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons**
[C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

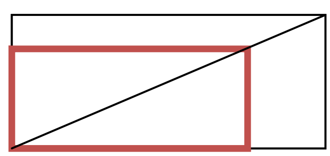
Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:
- Support students' understanding of comparison of corresponding angles with concrete materials.
- Promote reasoning strategies by having students prove that polygons have equal corresponding angles and proportional sides. Polygons of differing levels of difficulty can be presented to extend student's thinking.
- Construct polygons on grid paper, and then copy the same polygon on to larger or smaller grid paper to create a similar figure.
- Facilitate constructions with technology by using the tools for enlarging and reducing figures e.g. on an overhead projector.
- Emphasize the importance of modeling the process when students present their reasoning.

Suggested Activities

1. Test rectangles for similarity by placing two rectangles on top of each other with the smaller one fitting into a corner of the larger one. If the diagonal of the larger rectangle is also the diagonal of the smaller rectangle, the rectangles are similar.



Note: This test is only approximate and is only as precise as our measurement. This could be an opening for a discussion about accuracy in measurement.

2. A baseball coach wants to have a diagram of a baseball diamond that is similar to a real baseball diamond. A real baseball diamond is a square with side lengths of 27.4m. Have students construct a model using a ratio of 1: 500.
3. Provide a set of polygons of the same shape (e.g. all 4 sided polygons or all triangles), with a variety of sizes and orientation. Have students sort them by identifying those that are similar. Similar polygon could be traced and colour coded. Have students provide a justification for why the polygons are similar.
4. Cut out of coloured card stock, various regular and irregular polygons and one or more matching similar polygons. Place in a container and have students draw one or more shape, and then find the student or student with the matching similar polygon or polygons.

Possible Models: grid paper, Power polygons™, rulers, protractor

SCO: **SS3: Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons**
[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

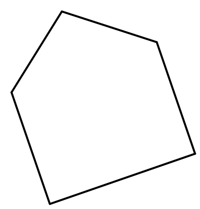
Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

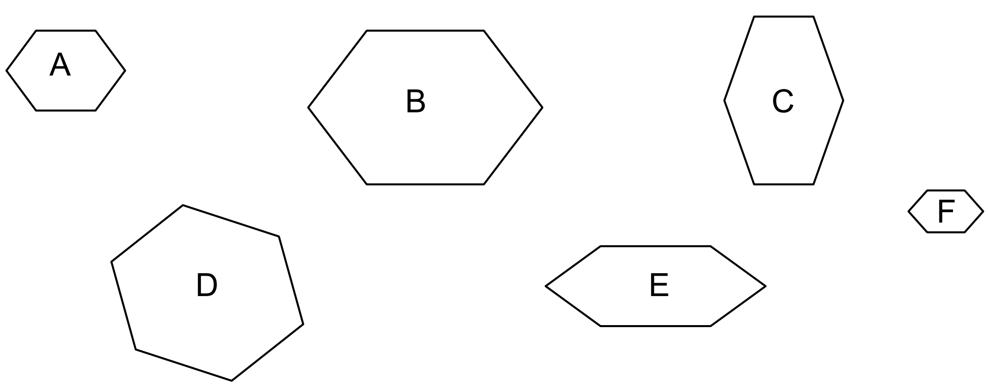
Whole Class/Group/Individual Assessment

1. Sketch a polygon that is similar to the given polygon below. Explain the criteria you have used to show that they are similar.



2. A photograph measuring 12.5 cm by 17.5 cm needs to be enlarged by a factor of one and a half. What will be the new dimensions of the photograph? Draw a diagram of both photographs to support your reasoning.

3. Determine which of the following polygons are similar. Justify your answer.



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
SS6 Demonstrate an understanding of tessellation by: explaining the properties of shapes that make tessellating possible; creating tessellations; identifying tessellations in the environment.	**SS4** Draw and interpret scale diagrams of 2-D shapes.	**M4** ***Geometry, Measurement and Finance 10*** Solve problems, using SI and Imperial systems, that involve area measurements of regular, composite and irregular 2-D shapes, including decimal and fractional measurements, and verify the solutions.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In Grade 8 students have explored proportions in relation to understanding tessellations. This will relate to this outcome as students explore scale diagrams of 2-D shapes.

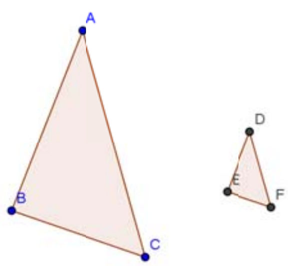
A **scale** is a comparison between an original diagram or object and a **scale diagram** which would be a drawing that is similar to the actual figure. These scale diagrams can be either an enlargement or reduction of the actual diagram depending on the context. If scale factors are bigger than 1, this will result in an enlargement whereas if the scale factor is less than 1, it is a reduction.

Students have been exposed to maps and pictures that have been drawn to scale and to images produced by photocopiers and computer software. The use of computer software can allow for a great deal of flexibility for the investigation of enlargement and reduction.

Graph paper, protractor, and rulers are tools to help students create images based on a given scale factor. Students could be asked to determine the scale factor given two similar images.

It should be noted that, when a ratio is used to represent an enlargement or reduction, the format of the ratio is *Scale Diagram: Original Diagram*. A ratio of 2:1 means the scale diagram is an enlargement to twice the original diagram. Likewise, a ratio of 1:3 means the model is a reduction to $\frac{1}{3}$ of the object, or the object was three times the model.

Students should understand the relationships between the corresponding sides of similar triangles. That is, if $\Delta ABC \sim \Delta DEF$ then the following ratios are equal $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$.



SCO: **SS4: Draw and interpret scale diagrams of 2-D shapes.**
[CN, R, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify an example in print and electronic media (e.g., newspapers, the Internet), of a scale diagram and interpret the scale factor.
- Draw a diagram to scale that represents an enlargement or reduction of a given 2-D shape.
- Determine the scale factor for a given diagram drawn to scale.
- Determine if a given diagram is proportional to the original 2-D shape and, if it is, state the scale factor.
- Solve a given problem that involves a scale diagram by apply the properties of similar triangles.

PLANNING FOR INSTRUCTION

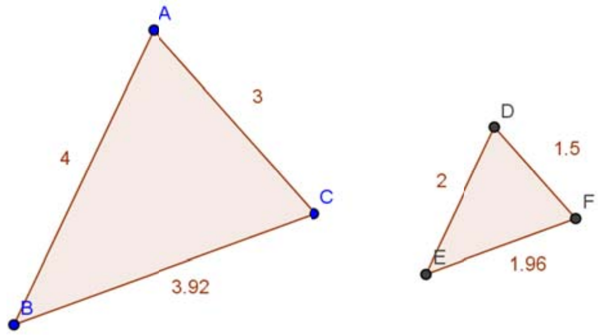
Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

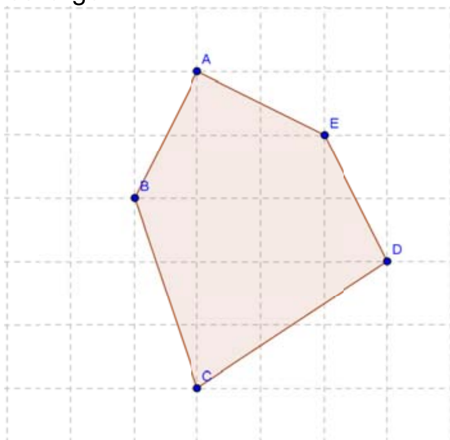
Consider the following strategies when planning lessons:

- Give students two triangles, and have them determine if they are similar. If they are, state the scale factor.



- Provide students with pictures of original and scale diagrams and have them determine the scale factor. For example, take a picture in class of a group of students and then measure the height of one of them. Have students determine the scale factor present, then use it to determine the heights of the others in the picture.

- Provide students with a 2-D shape on a graph paper and ask them to come up with a procedure to either reduce or enlarge the diagram. For example, give students the shape below and ask them to redraw the shape to be 3 times larger.



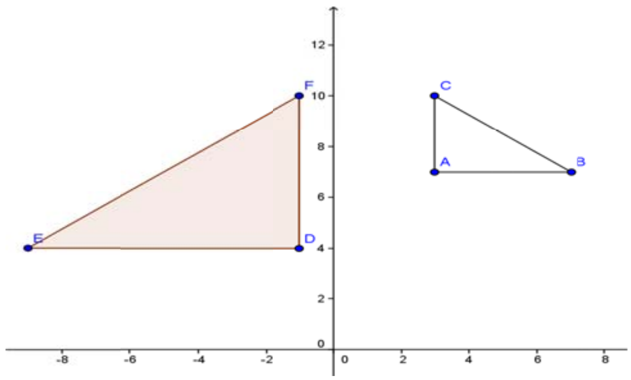
Suggested Activities

1. Use a chart as a useful way to reinforce the use of scale factor.

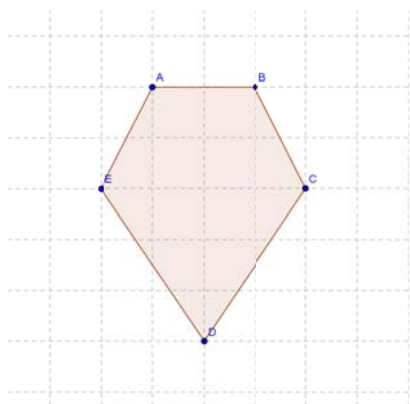
Size of Scale Diagram	Scale factor (scale size: actual size)	Size of Original Diagram
160 cm	?	40 cm
?	2:1 000 000	150 km
3 m	?	54 m
Chart can be expanded		

2. Given the following list of points to form two triangles, have students determine if they are similar.

   Triangle ΔABC with $A(3, 7)$ $B(7, 7)$ and $C(3,10)$
   Triangle ΔDEF with $D(-1, 4)$ $E(-9, 4)$ and $F(-1, 10)$.



3. Have students create an enlargement or reduction of the following shape (various shapes could be used).



4. Bring an example of a scale diagram in order to identify the size of the original object or diagram using the scale.

Possible Models: computer software programs such as Geometer's Sketchpad or websites such as GeoGebra (www.geogebra.org), graph paper, maps, blueprints, floor plans.

ASSESSMENT STRATEGIES

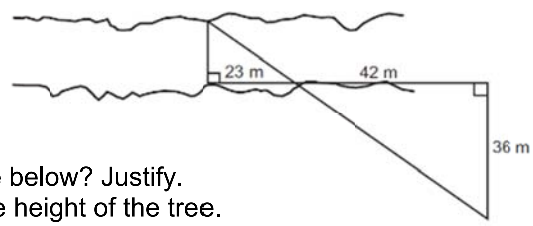
Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

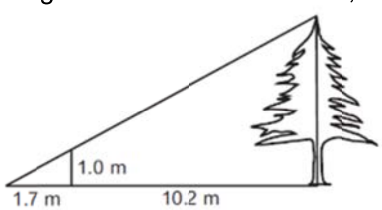
Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. Are the two triangles in the diagram below similar? Justify.
   If enough information is available, find the width of the river. If there is not enough information, what other information is needed?



2. Are the two triangles similar in the figure below? Justify.
   If enough information is available, find the height of the tree.



3. Using a map of New Brunswick and the scale factor determine distances from place to place. This could be done using the complete map or regional maps that can be found on the site http://www.new-brunswick.net/new-brunswick/maps/nb/nbmap.html A sample of the complete map is shown below.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS5: Demonstrate an understanding of line and rotational symmetry. [C, CN, PS, V]**			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
SS6 Demonstrate an understanding of tessellations by: explaining the properties of shapes that make tessellating possible; creating tessellations; identifying tessellations in the environment.	**SS5** Demonstrate an understanding of line and rotational symmetry.	

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In previous grades, students have performed transformations of 2-D shapes and have created and identified tessellations. Knowledge of this will be extended to **line symmetry** and **rotational symmetry*.*** Line symmetry is a line that divides a figure into two reflected parts. Shapes can have no line of symmetry or multiple lines of symmetry; these can exist in any orientation (vertical, horizontal, slanted). Rotation symmetry occurs when a figure can be turned about its centre so that it fits in its original outline. The **order of rotation** is the number of times a figure fits onto itself in one complete turn. The **angle of rotation** is the minimum angle required to turn a figure onto itself.

Paper folding and the Mira (transparent mirror) are encouraged in working with symmetry. When using paper folding, students can fold the shape onto itself to find the line(s) of symmetry. The Mira can be placed on the shape and when the reflection appears, the Mira defines the line of symmetry.

Rotations should be done by inspection if the turn centre is inside the figure or on a vertex. Some students may still require tracing paper in both situations; however, they should be encouraged to visualize the image before the transformation is performed. Similarly, when working with rotational symmetry, some students may choose to use tracing paper in order to rotate the shape about its centre to find the order and angle of rotation. This topic can provide an avenue for students to demonstrate their creativity.

SCO: **SS5: Demonstrate an understanding of line and rotational symmetry.**
[C, CN, PS, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Classify a given set of 2-D shapes or designs according to the number of lines of symmetry.
- Complete a 2-D shape or design given one half of the shape or design and a line of symmetry.
- Determine if a given 2-D shape or design has rotational symmetry about the point at the centre of the shape or design and, if it does, state the order and angle of rotation.
- Rotate a given 2-D shape about a vertex and draw the resulting image.
- Identify a line of symmetry or the order and angle of rotation symmetry in a given tessellation.
- Identify the type of symmetry that arises from a given transformation on the Cartesian plane.
- Complete, concretely or pictorially, a given transformation of a 2-D shape on a Cartesian plane, record the coordinates and describe the type of symmetry that results.
- Identify and describe the types of symmetry created in a given piece of artwork.
- Determine whether or not two given 2-D shapes on the Cartesian plane are related by either rotational or line symmetry.
- Draw, on a Cartesian plane, the translation image of a given shape using a given translation rule, such as R2, U3 or → → ↑ ↑ ↑, label each vertex and its corresponding ordered pair and describe why the translation does not result in line or rotational symmetry.
- Create or provide a piece of artwork that demonstrates line and rotational symmetry, and identify the line(s) of symmetry and the order and angle of rotation.

SCO: **SS5: Demonstrate an understanding of line and rotational symmetry.**
[C, CN, PS, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with or have them bring in a multitude of 2-D shapes and classify them according to the number of lines of symmetry, and rotational symmetry with the angle and order of rotation.
- Provide students with or have them bring in tessellations, artwork or wallpaper designs to identify lines of symmetry or the order and angle of rotation symmetry. This would be a great cross curricular activity with art classes. The designs produced can make interesting wall hangings for the classroom.
- Use graph paper and ask students to draw a shape and to cut it along a line of symmetry. Students exchange their drawing with another student who will complete the 2-D shape. Students should approach this by counting the spaces from the vertices to the line of symmetry in order to place each of the mirrored vertices and complete the shape.
- Ensure that students are exposed to a variety of shapes when looking at translations of 2-D shapes on the Cartesian plane, in order to recognize that translations often do not result in line or rotational symmetry.
- The works of M.C. Esher would make an interesting research project using the Internet. Another avenue to research could be Islamic Art which is often geometry-based and expresses the logic and order inherent in the Islamic vision of the universe.
- Wallpaper is a good source of designs which utilize transformational geometry and Esher-like transformations. If there is a wallpaper store close by, teachers can request old wallpaper books from discontinued designs. Students can look at the designs to find evidence of translations, reflections, and rotations, and record the transformations they observe. Many wallpaper designs incorporate multiple transformations, and some include interesting tessellations.

Suggested Activities:

1. On a Cartesian plane, have students,
   a) Sketch a quadrilateral.
   b) Label and record the coordinates of its vertices.
   c) Translate the quadrilateral [3R, 2U].
   d) Label and record the coordinates of the corresponding vertices of the image.
   e) Determine whether the shapes are related by either rotational or line symmetry and describe why.
2. Ask students to draw the following 2-D shapes: triangles (scalene, isosceles and equilateral), quadrilaterals (square, rectangle, parallelogram, trapezoid, rhombus) and other regular polygons (pentagon, hexagon, heptagon, octagon). Classify according to the number of lines of symmetry. Determine whether the shape has rotational symmetry and if it does, state the order and angle of rotation.
3. Give students a 2-D shape from the previous activity, and have them rotate the shape about a vertex and draw the resulting image.

Possible Models: Cartesian plane, Mira, artwork

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

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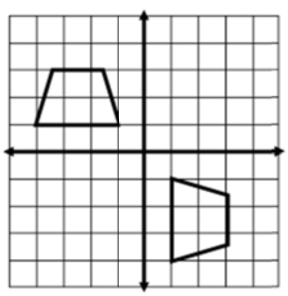
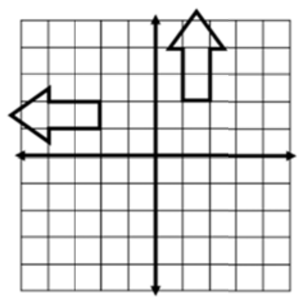
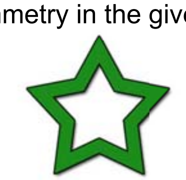
Whole Class/Group/Individual Assessment

1. Determine whether or not these pairs of shapes are related by line or rotation symmetry.

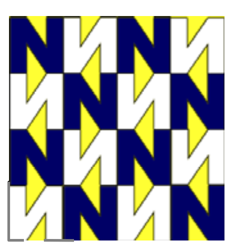
a) 

b) 

2. Give the number of lines of symmetry in the given figure below, and determine the order and angle of rotation.



3. Does the following tessellation have line symmetry, rotation symmetry, both or neither? Explain by describing the line of symmetry and/or the centre of rotation. If there is no symmetry, describe what changes would make the image symmetrical.



FOLLOW-UP ON ASSESSMENT

Guiding Questions

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SP1: Describe the effect of the following on data collection:** • **bias** • **use of language** • **ethics** • **cost** • **time and timing** • **privacy** • **cultural sensitivity** **[C, R, T, V]**			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
SP1 Critique ways in which data is presented.	**SP1** Describe the effect of the following on data collection: • bias • use of language • ethics • cost • time and timing • privacy • cultural sensitivity	

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In grade 8, students learned a variety of ways in which to present data (bar graphs, line graphs, circle graphs, pictographs), and to evaluate their strengths and limitations. Terms such as discrete and continuous data, accuracy, choice of intervals, and trends were reinforced. Students learned how to justify their conclusions and identify inconsistent and misrepresented data.

In grade 9, students will continue to develop data analysis and focus on factors that affect the collection of data.

Students will learn, through case studies, how the presentation of the data influences public perception. Those influences may include bias, ethics, cultural sensitivity, and the effects of language, cost, time, timing and privacy.

Students should provide examples to illustrate how bias, use of language, ethics, cost, time, timing, privacy and/or cultural sensitivity may influence data.

SCO: **SP1: Describe the effect of the following on data collection:**

- **bias**
- **use of language**
- **ethics**
- **cost**
- **time and timing**
- **privacy**
- **cultural sensitivity**

[C, R, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Analyze a given case study of data collection and identify potential problems related to bias, use of language, ethics, cost, time, timing, privacy and/or cultural sensitivity.
- Provide examples to illustrate how bias, use of language, ethics, cost, time and timing, privacy or cultural sensitivity may influence the data.

SCO: **SP1: Describe the effect of the following on data collection:**
- **bias**
- **use of language**
- **ethics**
- **cost**
- **time and timing**
- **privacy**
- **cultural sensitivity**

[C, R, T, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:
- Use graphs from various sources (newspapers, magazines, online etc.), and computer spreadsheet applications (e.g., Microsoft Excel), and websites such as Statistics Canada (http://www40.statcan.gc.ca/z01/cs0002-eng.htm).

Suggested Activities

1. Tell students that based on the survey completed by Mac World last month, 90% of the population prefers Apple over PC computers.

   Discuss the possible effect of the following data collection:
   - Bias: How many people were surveyed? What was the age range of the population? What percentage of those surveyed were Mac users?
   - Use of language: Is the question clear, and not leading to a particular response?
   - Ethics: Will the results be used for appropriate purposes originally given?
   - Cost: Does the cost of the study outweigh the benefits?
   - Time: Is the timing of, and the time required to do the survey, appropriate?
   - Privacy: Are the results confidential?
   - Cultural sensitivity: Are the questions offensive to a particular culture?
2. Have students, in groups, develop survey questions such as:
   a) Since students have many after school activities, do you think teachers should assign less homework?
   b) Do you agree that the price of concert tickets is too high?

   Ask students to identify the potential problems associated with the questions asked.
3. Compile some data examples (good/bad) from math texts from previous years for analysis and discussion.
4. Investigate websites that are misleading (e.g. the endangered tree octopus, the house hippo) and discuss the value of critical thinking skills.

Possible Models:

SCO: **SP1: Describe the effect of the following on data collection:**

- **bias**
- **use of language**
- **ethics**
- **cost**
- **time and timing**
- **privacy**
- **cultural sensitivity**

[C, R, T, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/One-on-One Assessment

1. For the following survey questions, identify the sources of bias and suggest ways to remove it.
   a) At a soccer game, a survey was given and the results showed that when asked to give their favourite sport, 85% of the youth responded it was soccer.
   b) Do you think that small dogs make good pets even though they are yappy?
   c) Create a biased question about the use of technology among teenagers.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SP2: Select and defend the choice of using either a population or a sample of a population to answer a question.**
[C, CN, PS, R]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
SP1 Critique ways in which data is presented.	**SP2** Select and defend the choice of using either a population or a sample of a population to answer a question.	

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students will need to understand the relationship between a population and a sample.

All of the individuals in the group being studied are called a **population**. For example the population in a federal election is all eligible voters, and when data is collected from each member of the population it is called a **census.**

Since it is often impractical to gather information about entire populations, sampling is a common statistical technique. Any group of individuals selected from the population would be referred to as a **sample**. For the example of a federal election a sample could be taken of 100 individuals chosen from each province or territory.

When a sample is representative of the population, the data collected from the sample leads to valid conclusions. Students will need to understand issues with respect to sampling strategies and sample size in order to properly draw inferences from sample data. By conducting experiments or simulations and examining the data collected, students should understand that larger sample sizes increase the likelihood that the statistical results will approximate expected values or population characteristics.

There are many different ways to select samples, with random samples the most likely to produce valid conclusions.

SCO: **SP2: Select and defend the choice of using either a population or a sample of a population to answer a question.**
[C, CN, PS, R]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify whether a given situation represents the use of a sample or a population.
- Provide an example of a situation in which a population may be used to answer a question and justify the choice.
- Provide an example of a question where a limitation precludes the use of a population and describe the limitation, e.g., too costly, not enough time, limited resources.
- Identify and critique a given example in which a generalization from a sample of a population may or may not be valid for the population.

SCO: **SP2: Select and defend the choice of using either a population or a sample of a population to answer a question.**
[C, CN, PS, R]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Consider the following two situations.

  In the first situation a television survey is done to find the fraction of the public that watches a particular television program on a particular night and to determine if it is watched by more women than men.

  In the second situation, a quality engineer must estimate what percentage of bottles rolling off an assembly line are defective.

  In both these situations, information is gathered about a large group of people or things. The expense of contacting every person or inspecting every bottle, and the time it would take, is formidable so information is gathered about only part of the group (a sample) in order to draw conclusions about the whole group (the population).
- Ensure students recognize that choosing a representative sample from a large and varied population can be a complex task. It is important to be clear about what population is to be described and exactly what is to be measured.

  Areas of concern
  - How can a sample be chosen so that it is truly representative of the population?
  - If a sample from a population differs from another sample from the same population, how confident can you be about predicting the true population percentage?
  - Does the size of the sample make a difference?

Suggested Activities

1. Use *Census at School* (http://www19.statcan.gc.ca/r000-eng.htm) to gather information on various topics on Canadian youth such as favourite subjects, height, weight, eye color, TV habits, etc. (*Census at School* offers students a golden opportunity to be involved in the collection and analysis of their own data and to experience what a census is like.)
2. Identify a population for a specific situation (e.g., a cell phone carrier wants to know which brand and model of phone the students are currently using) and state whether the whole population or a sample of the population should be sampled. Explain your reasoning.

Possible Models: Computer and internet access

SCO: **SP2: Select and defend the choice of using either a population or a sample of a population to answer a question.**
[C, CN, PS, R]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. Identify the population for each situation given below, and indicate if you would survey the population or just a sample.
   a) Vito's restaurant would like to know which lunch menu item its customers would prefer.
   b) Bell Canada would like to know how many of its customers would like to have caller id as a feature.
   c) Health Canada would like to find out reasons why some Canadians chose not to get the H1N1 vaccine.

2. For the following scenarios, what problems can you see with the generalizations that were made?
   a) At the school cafeteria, employees conducted a survey about what snacks would be offered at break time during the school day. The cafeteria worker handed out a survey to every fourth person who came through the line on a particular day and gathered the data from these. It was concluded from this that students would like to see more granola bars offered during the breaks.
   b) Student council surveyed students about how best to spend the activities budget for the coming year. It randomly surveyed students at a soccer game. Student council concluded more money should be spent on athletic teams.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SP3: Construct, label, and interpret histograms to solve problems. [C, CN, PS, R, T, V]**			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Grade Eight	Grade Nine	Grade Ten
	SP3 Construct, label, and interpret histograms to solve problems.	.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Histograms are similar to bar graphs, except that histograms express continuous data and bar graphs express discrete data (see examples of a bar graph and histogram below). Students have learned in previous grades as well as in PR1, that continuous data is different from discrete data.

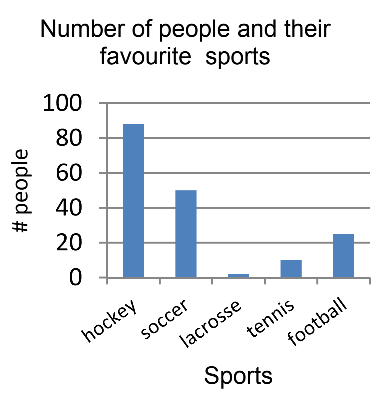
The first step in creating a histogram is to group a continuous sequence of numerical data (e.g., ages, time, heights, percentages, etc.) into appropriate **intervals** or **classes**. The data is arranged in a **frequency table**. The **range** of the interval is used to determine the width of the bars which should be neither too narrow nor too broad. Intervals are typically the same size and the number of intervals is usually kept to between 4 and 10

For histograms, the values on the x-axis (and x- column) indicate the boundaries of the intervals. A number at the lower boundary will be included but a number at the upper boundary will not be included. For example, in the histogram graph below, for ages 10-20, the data for a concert-goer who was 19 would be included, but a 20-year old would be in the 20-30 group.

When drawing the histogram, the height of each bar is determined by the number of pieces of data that are included in the interval. Since the data in histograms is continuous, there are no spaces between the bars. All data must included. If there is an interval that contains no pieces of data, a space is left where that bar would have been drawn.

Like other data displays, all histograms must include a meaningful **title**, **labels for the axes**, and **values** to indicate the intervals of the data. The axes labels should include units where applicable (e.g years, seconds, centimetres, percentages). The values for the intervals should be displayed at the limits of the bars.

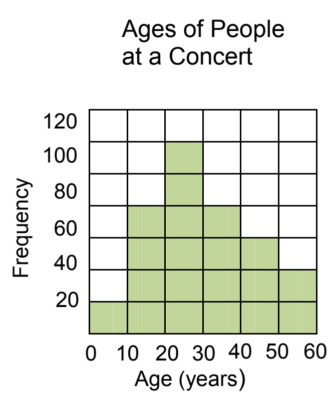
Bar Graph



Histogram

Frequency Table

Ages	Frequency
0-10	20
10-20	80
20-30	120
30-40	80
40-50	60
50-60	40



SCO: **SP3: Construct, label, and interpret histograms to solve problems.**
[C, CN, PS, R, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify common attributes of histograms: title, labels for the axes, values to indicate the intervals of the data, and no spaces between bars.
- Create a histogram from a set of data with appropriate scale and intervals, title, and labels.
- Determine whether a given set of data can be represented by a histogram (continuous data) or a bar graph (discrete data) and explain why.
- Interpret a histogram to answer questions and draw conclusions.

SCO: **SP3: Construct, label, and interpret histograms to solve problems. [C, CN, PS, R, T, V]**

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:
- Provide students with several examples of bar graphs and histograms to explore the similarities and differences between the two types of displays.
- Use websites such as Statistics Canada (www.statcan.gc.ca) for background information, data and lesson plans; or NCTM's Illumination activities (http://illuminations.nctm.org/ActivityDetail.aspx?ID=78).
- Use computer applications (e.g., Microsoft Excel or *TinkerPlots*® http://www.keypress.com/x5715.xml) and graphing calculators (TI-83) to allow students to explore various data displays and be able to quickly modify the display without redrawing it manually.
- Have students examine the shape of various histograms and discuss what information this tells them.
- Ensure that data used in the investigations of this outcome is relevant to students.
- Explore opportunities in other subject areas such as Science or Social Studies to apply students' knowledge of histograms.

Suggested Activities

1. Provide students with a set of data that has already been organized into a frequency table such as the one on the right which shows the number of pedestrians killed in one year in a large city by age. Have students construct the histogram, including a title and axis labels and then discuss the shape of the distribution of the data and possible reasons for it.

Frequency Table

Ages	Frequency
0-10	84
15-20	28
20-30	9
30-40	24
40-50	19
50-60	43
60-70	63
70-80	58

2. Provide students with two sets of data and have them determine the type of graph they would use to display it, and explain the choice. Construct the two graphs.

Population by age group

Age Group (years)	*Number of Canadians*
0-15	3 637 000
15-30	4 226 700
30-45	4 669 300
45-60	4 564 700

Population by Atlantic Province

Province	*Population in 2009*
Newfoundland and Labrador	508 900
Prince Edward Island	141 000
Nova Scotia	938 200
New Brunswick	749 500

3. Have students research a topic of personal interest that could be displayed in a histogram. Collect the data and determine what the intervals should be (try to have less than 10 groups). Construct the histogram including a meaningful title and all required labels.

Possible Models: graphs from various sources (newspapers, magazines, etc.)

SCO: **SP3: Construct, label, and interpret histograms to solve problems.**
[C, CN, PS, R, T, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

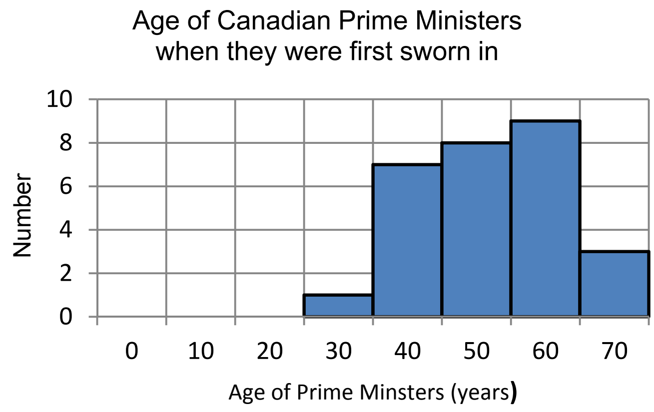
Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/One-on-One Assessment

1. Have students explain when they would use a bar graph and when they would use a histogram to display a given set of data. Provide examples, such as the goals scored by individual players on a team or the number of goals scored by the team for each game during the past 6 weeks.
2. In December the number of hours of bright sunshine recorded at 36 selected stations was as follows:

   16 25 41 20 35 20 16 8 38 23 25 38 41 34 24 39 47 45
   17 42 44 47 45 51 35 37 51 39 14 14 40 44 50 40 31 22

   Have students:
   a) choose an interval and create a frequency table for the data.
   b) use the grouped data to create a histogram (ensure that it includes a title and all labels).
   c) choose a different interval and repeat a) and b).
   d) compare the two histograms and explain which they feel is more useful.
3. Have students examine and analyze the data expressed on the histogram shown below.



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FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SP4: Develop and implement a project plan for the collection, display and analysis of data by:**
- **formulating a question for investigation**
- **choosing a data collection method that includes**
- **social considerations**
- **selecting a population or a sample**
- **collecting the data**
- **displaying the collected data in an appropriate manner**
- **drawing conclusions to answer the question.**

[C, PS, R, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
SP1 Critique ways in which data is presented.	**SP4** Develop and implement a project plan for the collection, display and analysis of data by: • formulating a question for investigation • choosing a data collection method that includes social considerations • selecting a population or a sample • collecting the data • displaying the collected data in an appropriate manner • drawing conclusions to answer the question.	

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In previous grades, students have been collecting, displaying and interpreting data represented in various tables and graphs. This project will consolidate prior knowledge and could be completed as an interdisciplinary or cross curricular assignment (science, language arts, health, etc.). Students can draw upon data that are available through such sources as Statistics Canada, as well as other government documents and reports.

Students will require guidance when developing a plan for their project. Models of data analysis projects should be reviewed with students so they can recognize what quality work looks like. The development of a relevant question will determine the success of the project so time will have to be devoted to assisting students' in their choice of a question.

SCO: **SP4: Develop and implement a project plan for the collection, display and analysis of data by:**

- **formulating a question for investigation**
- **choosing a data collection method that includes**
- **social considerations**
- **selecting a population or a sample**
- **collecting the data**
- **displaying the collected data in an appropriate manner**
- **drawing conclusions to answer the question.**

[C, PS, R, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Create a rubric to assess a project that includes the assessment of:
  - a question for investigation
  - the choice of a data collection method that includes social considerations
  - the selection of a population or a sample and justifying the choice
  - the display of the collected data
  - the conclusions to answer the question.
- Develop a project plan that describes:
  - a question for investigation
  - the method of data collection that includes social considerations
  - the method for selecting a population or a sample
  - the method to be used for collection of the data
  - the methods for analysis and display of the data.
- Complete the project according to the plan, draw conclusions and communicate findings to an audience.
- Self-assess the completed project by appling the rubric.

SCO: **SP4: Develop and implement a project plan for the collection, display and analysis of data by:**
- **formulating a question for investigation**
- **choosing a data collection method that includes**
- **social considerations**
- **selecting a population or a sample**
- **collecting the data**
- **displaying the collected data in an appropriate manner**
- **drawing conclusions to answer the question.**

[C, PS, R, T, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:
- Create a plan with the various components of a data management project: the question that will be investigated, methods of data collection, sampling procedures, data collection, data displays, and conclusions.
- Complete statistics projects in pairs or small groups. Students could assess their projects in pairs using the rubric that they have previously created.
- Consider using technology to create data displays.

Suggested Activities

1. Include self-assessment for identifying areas for improvement and identification of strengths and needs.
2. Brainstorm to explore possible questions for investigation.
3. Explore and critique rubrics that have been developed so students are exposed to models.

Possible Models:

SCO: **SP4: Develop and implement a project plan for the collection, display and analysis of data by:**
- **formulating a question for investigation**
- **choosing a data collection method that includes**
- **social considerations**
- **selecting a population or a sample**
- **collecting the data**
- **displaying the collected data in an appropriate manner**
- **drawing conclusions to answer the question.**

[C, PS, R, T, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. Rubric development may be a new concept to students at this grade level. A rubric should be developed prior to the beginning of the project to clarify exactly what is expected and how the project will be assessed. A rubric should include the criteria that will be assessed and a description of each level of performance. Rubric development could be completed as a class activity. An example is shown below.

Criteria	Level 1	Level 2	Level 3	Level 4
a question for investigation				
the choice of a data collection method that includes social considerations				
the selection of a population or a sample and justifying the choice				
the display of the collected data				
conclusions to answer the question.				

2. Use the rubric to self-assess student work before completing the final draft and use the rubric to improve the work before a grade is assigned.
3. Present projects to the class and use rubrics to assess the final product.
4. Present projects in a math fair or as a display during a parent night.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SP5: Demonstrate an understanding of the role of probability in society.** **[C, CN, R, T]**			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Eight	Grade Nine	Grade Ten
SP2 Solve problems involving the probability of independent events.	**SP4** Demonstrate an understanding of the role of probability in society.	

ELABORATION

Guiding Questions:
• What do I want my students to learn?
• What do I want my students to understand and be able to do?

Students entering Grade 9 should understand the difference between theoretical and experimental probability, and should be able to express probabilities for single and independent events in fractions, percents and decimals.

In grade 9, the focus of study is for students to understand the role that probability plays in society by looking at the probability of events occurring and by examining decisions that are based on those predictions. Students should be exposed to a variety of examples from daily life in which probability is used. Some examples are:

- Insurance premiums that are established based on the historical data of a certain gender, age group or region making claims.
- Warranty periods based on probable lifespan of a product
- Number of units manufactured based on probable units that will sell
- Projecting of winners of an election from past data
- Determining probability of experiencing the side effects of a drug
- Developing a schedule of flights and crew, and setting fares based on the probability of demand at different times of year
- Weather forecasts and the probability of precipitation and other weather systems

The work of students should be focused on situations which are familiar to them. Discussions about how predictions are made (a mix of theoretical and experimental probability and subjective judgement) should be a focus. Students will quickly realize that experimental probability is the one most widely used when making predictions. The types of **assumptions** made when making these predictions should also be addressed.

SCO: **SP5: Demonstrate an understanding of the role of probability in society.**
[C, CN, R, T]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Provide an example from print and electronic media, e.g., newspapers, the internet, where probability is used and justify how this example makes use of probability.
- Identify the assumptions associated with a given probability and explain the limitations of each assumption.
- Explain how a single probability can be used to support opposing positions.
- Explain, using examples, how decisions based on probability may be a combination of theoretical probability, experimental probability and subjective judgment.

SCO: **SP5: Demonstrate an understanding of the role of probability in society. [C, CN, R, T]**

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Give students the opportunity to explore the media to find examples of predictions based on probability in everyday life.
- Give students the opportunity to explore decision-making based on probability. They should use a sample to determine the probability of an event, use the results and subjective judgment to make predictions and explain the reasonableness of the predictions, based on any assumptions that they made. If at all possible, the reasonableness of the predictions should be tested.
- As a class, look for examples, as a class, in the media where probability is used to support or reject a position.

Suggested Activities

1. Use the sampling from SP3 to:
   - Use the results to make predictions about the general population.
   - Identify the assumptions made and the limitations of these assumptions.
   - Discuss how much reliance there was on theoretical probability, experimental probability and subjective judgement to make that prediction.

Possible Models:

SCO: **SP5: Demonstrate an understanding of the role of probability in society. [C, CN, R, T]**

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and as individuals. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

1. Access www.climate.weatheroffice.ec.gc.ca/ClimateData/canada_e.html and search for data about your hometown to make predictions for the current month (precipitation amounts, mean temperature, etc). Discuss any assumptions you may have had when making these predictions and explain the limitations of these assumptions.

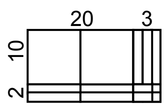
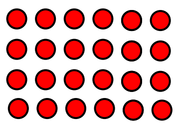
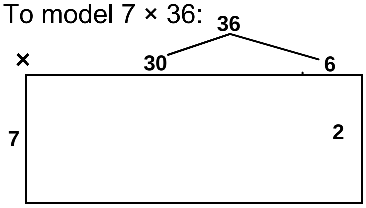
2. For your school determine the probably number of students who will go on to post-secondary education next year. Think about various ways to determine this probability. For example, use school data such as a request for transcripts from previous years, or approach the Awards Committee to see how many scholarships were obtained.

FOLLOW-UP ON ASSESSMENT

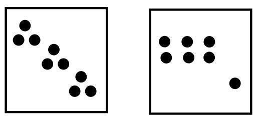
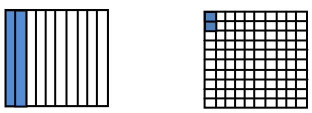
Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

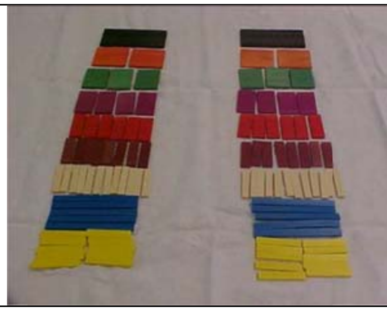
GLOSSARY OF MODELS

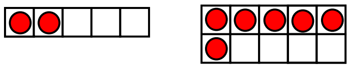
This glossary is identical for Grades K-9. Most of the models have a variety of uses at different grade levels. More information as to which models can be used to develop specific curriculum outcomes is located on the *Instructional Strategies* section of each four-page spread in this curriculum document. The purpose of this glossary is to provide a visual and a brief description of each model.

Name	Picture	Description
Algebra tiles		• Sets include "X" tiles (rectangles), "X^2" tiles (large squares), and integer tiles (small squares). • All tiles have a different colour on each side to represent positive and negative. Typically the "X" tiles are green and white and the smaller squares are red and white. • Some sets also include "Y" sets of tiles which are a different colour and size than the "X" tiles.
Area Model	To model 12 × 23:  	• Use base ten blocks to represent the parts of each number that is being multiplied. • To find the answer for the example shown, students can add the various parts of the model: 200 + 30 + 40 + 6 = 276. • This model can also be used for fraction multiplication.
Arrays and Open Arrays	To model 4 × 6:  To model 7 × 36:  	• Use counters arranged in equal rows or columns or a Blackline Master with rows and columns of dots. • Helpful in developing understanding of multiplication facts. • Grids can also be used to model arrays. • Open arrays allows students to think in amounts that are comfortable for them and does not lock them into thinking using a specific amount. These arrays help visualize repeated addition and partitioning and ultimately using the distributive property.
Attribute Blocks		• Sets of blocks that vary in their attributes: o 5 shapes • circle, triangle, square, hexagon, rectangle o 2 thicknesses o 2 sizes o 3 colours
Balance (pan or beam) scales		• Available in a variety of styles and precision. • Pan balances have a pan or platform on each side to compare two unknown amounts or represent equality. Weights can be used on one side to measure in standard units. • Beam balances have parallel beams with a piece that is moved on each beam to determine the mass of the object on the scale. Offer greater accuracy than a pan balance.

<table>
<tr><td>Base Ten Blocks</td><td></td><td>• Include unit cubes, rods, flats, and large cubes.
• Available in a variety of colours and materials (plastic, wood, foam).
• Usually 3-D.</td></tr>
<tr><td>Beam Balance</td><td colspan="2">☞ see Balance (pan or beam)</td></tr>
<tr><td>Carroll Diagram</td><td>Example:<table><tr><td></td><td>1-digit</td><td>2-digit</td></tr><tr><td>Even</td><td>2, 4, 6, 8</td><td>26, 34</td></tr><tr><td>Odd</td><td>1, 3, 5, 7, 9</td><td>15, 21</td></tr></table></td><td>• Used for classification of different attributes.
• The table shows the four possible combinations for the two attributes.
• Similar to a Venn Diagram.</td></tr>
<tr><td>Colour Tiles</td><td></td><td>• Square tiles in 4 colours (red, yellow, green, blue)
• Available in a variety of materials (plastic, wood, foam).</td></tr>
<tr><td>Counters (two colour)</td><td></td><td>• Counters have a different colour on each side.
• Available in a variety of colour combinations, but usually are red & white or red & yellow.
• Available in different shapes (circles, squares, bean).</td></tr>
<tr><td>Cubes (Linking)</td><td></td><td>• Set of interlocking 2 cm cubes.
• Most connect on all sides.
• Available in a wide variety of colours (usually 10 colours in each set).
• Brand names include: Multilink, Hex-a-Link, Cube-A-Link.
• Some types only connect on two sides (brand name example: Unifix).</td></tr>
<tr><td>Cuisenaire Rods®</td><td></td><td>• Set includes 10 different colours of rods.
• Each colour represents a different length and can represent different number values or units of measurement.
• Usual set includes 74 rods (22 white, 12 red, 10 light green, 6 purple, 4 yellow, 4 dark green, 4 black, 4 brown, 4 blue, 4 orange).
• Available in plastic or wood.</td></tr>
</table>

Dice (Number Cubes)		• Standard type is a cube with numbers or dots from 1 to 6 (number cubes). • Cubes can have different symbols or words. • Also available in: ◦ 4-sided (tetrahedral dice) ◦ 8-sided (octahedral dice) ◦ 10-sided (decahedra dice) ◦ 12-sided, 20-sided, and higher ◦ Place value dice
Dominoes		• Rectangular tiles divided in two-halves. • Each half shows a number of dots: 0 to 6 or • 0 to 9. • Sets include tiles with all the possible number combinations for that set. • Double-six sets include 28 dominoes. • Double-nine sets include 56 dominoes.
Dot Cards		• Sets of cards that display different number of dots (1 to 10) in a variety of arrangements. • Available as free Blackline Master online on the "Teaching Student-Centered Mathematics K-3" website (BLM 3-8).
Decimal Squares®		• Tenths and hundredths grids that are manufactured with parts of the grids shaded. • Can substitute a Blackline Master and create your own class set.
Double Number Line	☞ see Number lines (standard, open, and double)	
Five-frames	☞ see Frames (five- and ten-)	
Fraction Blocks		• Also known as Fraction Pattern blocks. • 4 types available: pink "double hexagon", black chevron, brown trapezoid, and purple triangle. • Use with basic pattern blocks to help study a wider range of denominators and fraction computation.
Fraction Circles		• Sets can include these fraction pieces: 1, $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$ • Each fraction graduation has its own colour. • It is helpful to use ones without the fractions marked on the pieces for greater flexibility (using different piece to represent 1 whole).

<table>
<tr><td>Fraction Pieces</td><td></td><td>

Rectangular pieces that can be used to represent the following fractions:
$\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$
Offers more flexibility as different pieces can be used to represent 1 whole.
Each fraction graduation has its own colour.
Sets available in different quantities of pieces.
</td></tr>
<tr><td>Frames
(five- and ten-)</td><td></td><td>

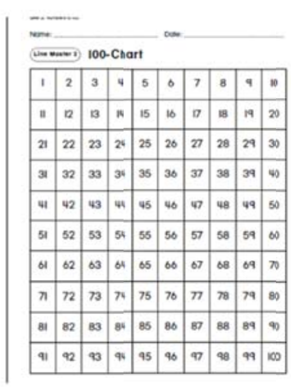
Available as a Blackline Master in many resources or you can create your own.
Use with any type of counter to fill in the frame as needed.
</td></tr>
<tr><td>Geoboards</td><td></td><td>

Available in a variety of sizes and styles.

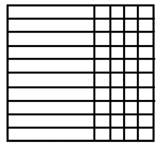
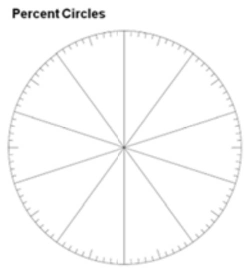
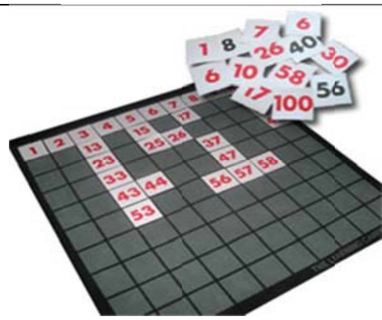
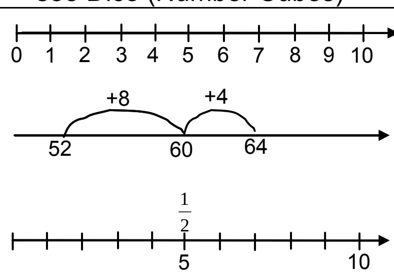
5 × 5 pins
11 × 11 pins
Circular 24 pin
Isometric

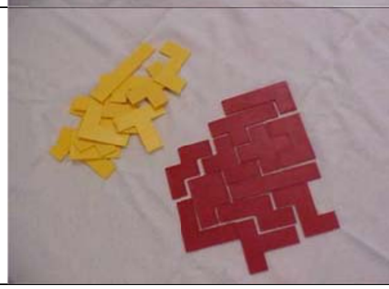
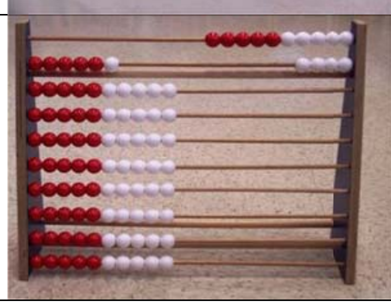
Clear plastic models can be used by teachers and students on an overhead.
Some models can be linked to increase the size of the grid.
</td></tr>
<tr><td>Geometric Solids</td><td></td><td>

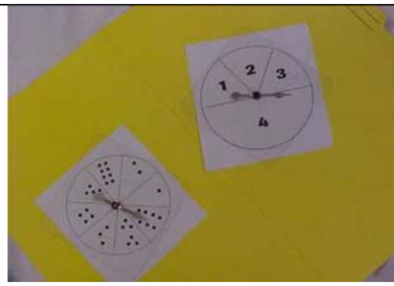
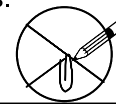
Sets typically include a variety of prisms, pyramids, cones, cylinders, and spheres.
The number of pieces in a set will vary.
Available in different materials (wood, plastic, foam) and different sizes.
</td></tr>
<tr><td>Geo-strips</td><td></td><td>

Plastic strips that can be fastened together with brass fasteners to form a variety of angles and geometric shapes.
Strips come in 5 different lengths. Each length is a different colour.
</td></tr>
<tr><td>Hundred Chart</td><td>
Name: Date:
Line Master 1 100-Chart
<table>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr>
<tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr>
<tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr>
<tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr>
<tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr>
<tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr>
<tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr>
<tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr>
<tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr>
</table></td><td>

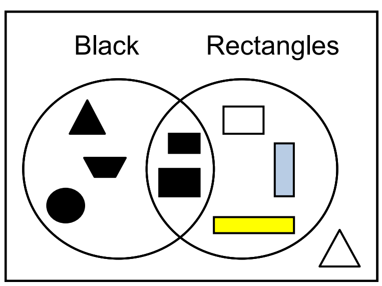
10 × 10 grid filled in with numbers 1-100 or 0 - 99.
Available as a Blackline Master in many resources or you can create your own.
Also available as wall charts or “Pocket” charts where cards with the numbers can be inserted or removed.
</td></tr>
</table>

Hundred Grid		• 10 × 10 grid. • Available as Blackline Master in many resources.
Hundredths Circle		• Circle divided into tenths and hundredths. • Also known as "percent circles".
Learning Carpet®	 http://www.thelearningcarpet.ca	• 10 × 10 grid printed on a floor rug that is six feet square. • Number cards and other accessories are available to use with the carpet.
Linking Cubes	☞ see Cubes (Linking)	
Mira®		• Clear red plastic with a bevelled edge that projects reflected image on the other side. • Other brand names include: Reflect-View and Math-Vu™.
Number Cubes	☞ see Dice (Number Cubes)	
Number Lines (standard, open, and double)		• Number lines can begin at 0 or extend in both directions. • Open number lines do not include pre-marked numbers or divisions. Students place these as needed. • Double number lines have numbers written above and below the line to show equivalence.
Open Arrays	☞ see Arrays and Open Arrays	
Open Number Lines	☞ see Number Lines (standard, open, and double)	
Pan Balance	☞ see Balance (pan or beam)	

Pattern Blocks		• Standard set includes: • Yellow hexagons, red trapezoids, blue parallelograms, green triangles, orange squares, beige parallelograms. • Available in a variety of materials (wood, plastic, foam).
Pentominoes		• Set includes 12 unique polygons. • Each is composed of 5 squares which share at least one side. • Available in 2-D and 3-D in a variety of colours.
Polydrons		• Geometric pieces snap together to build various geometric solids as well as their nets. • Pieces are available in a variety of shapes, colours, and sizes: ▪ Equilateral triangles, isosceles triangles, right-angle triangles, squares, rectangles, pentagons, hexagons • Also available as Frameworks (open centres) that work with Polydrons and another brand called G-O-Frames™.
Power Polygons™		• Set includes the 6 basic pattern block shapes plus 9 related shapes. • Shapes are identified by letter and colour.
Rekenrek		• Counting frame that has 10 beads on each bar: 5 white and 5 red. • Available with different number of bars (1, 2, or 10).

Spinners		• Create your own or use manufactured ones that are available in a wide variety:
		  ◦ number of sections;
		  ◦ colours or numbers;
		  ◦ different size sections;
		  ◦ blank.
		• Simple and effective version can be made with a pencil held at the centre of the spinner with a paperclip as the part that spins.
		

Tangrams		• Set of 7 shapes (commonly plastic):
		  ◦ 2 large right-angle triangles
		  ◦ 1 medium right-angle triangle
		  ◦ 2 small right-angle triangles
		  ◦ 1 parallelogram
		  ◦ 1 square
		• 7-pieces form a square as well as a number of other shapes.
		• Templates also available to make sets.

Ten-frames	☞ see Frames (five- and ten-)	•
Trundle Wheel		• Tool for measuring longer distances. • Each revolution equals 1 metre usually noted with a click.
Two Colour Counters	☞ see Counters (two colour)	
Venn Diagram		• Used for classification of different attributes. • Can be one, two, or three circles depending on the number of attributes being considered. • Attributes that are common to each group are placed in the interlocking section. • Attributes that don't belong are placed outside of the circle(s), but inside the rectangle. • Be sure to draw a rectangle around the circle(s) to show the "universe" of all items being sorted. • Similar to a Carroll Diagram.

List of Grade 9 Specific Curriculum Outcomes

Number (N)

1. Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by: representing repeated multiplication using powers; using patterns to show that a power with an exponent of zero is equal to one; solving problems involving powers.
2. Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents.
3. Demonstrate an understanding of rational numbers by: comparing and ordering rational numbers; solving problems that involve arithmetic operations on rational numbers.
4. Explain and apply the order of operations, including exponents, with and without technology.
5. Determine the square root of positive rational numbers that are perfect squares.
6. Determine an approximate square root of positive rational numbers that are non-perfect squares.

Patterns & Relations (PR)

(Patterns)

1. Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.
2. Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.

(Variables and Equations)

3. Model and solve problems using linear equations of the form:

   $ax = b$; $\frac{x}{a} = b$, $a \neq 0$; $ax + b = c$; $\frac{x}{a} + b = c$, $a \neq 0$; $\frac{a}{x} = b$, $x \neq 0$

   $ax + b = cx + d$; $a(bx + c) = d(ex + f)$; $a(x + b) = c$; $ax = b + cx$

   concretely, pictorially and symbolically, where *a*, *b*, *c, d, e, and f* are rational numbers.
4. Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.
5. Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).
6. Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2).
7. Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically.

Shape and Space (SS)

(Measurement)

1. Solve problems and justify the solution strategy using circle properties, including: the perpendicular from the centre of a circle to a chord bisects the chord; the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc; the inscribed angles subtended by the same arc are congruent; a tangent to a circle is perpendicular to the radius at the point of tangency.

(3-D Objects and 2-D Shapes)

2. Determine the surface area of composite 3-D objects to solve problems.
3. Demonstrate an understanding of similarity of polygons.

(Transformations)

4. Draw and interpret scale diagrams of 2-D shapes.
5. Demonstrate an understanding of line and rotation symmetry.

Statistics and Probability (SP)

(Data Analysis)

1. Describe the effect of: bias; use of language; ethics; cost; time and timing; privacy; cultural sensitivity on the collection of data.
2. Select and defend the choice of using either a population or a sample of a population to answer a question.
3. Construct, label, and interpret histograms to solve problems.
4. Develop and implement a project plan for the collection, display and analysis of data by: formulating a question for investigation; choosing a data collection method that includes social considerations; selecting a population or a sample; collecting the data; displaying the collected data in an appropriate manner drawing conclusions to answer the question.

(Chance and Uncertainty)

5. Demonstrate an understanding of the role of probability in society.

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