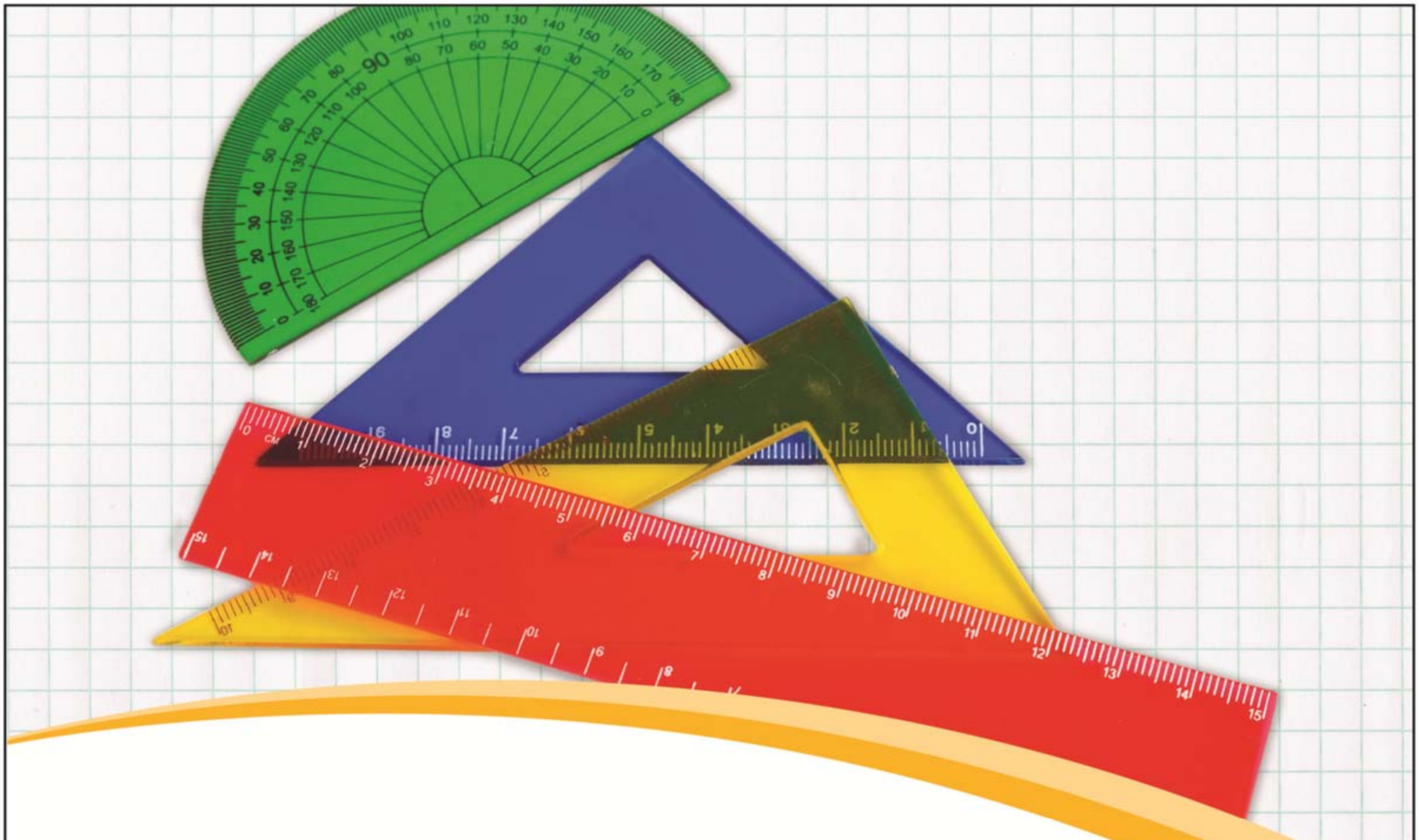


Mathematics Grade 6 Curriculum

Implemented September 2010

New Nouveau Brunswick

Acknowledgements

The Department of Education of New Brunswick gratefully acknowledges the contributions of the following groups and individuals toward the development of the *New Brunswick Grade 3 Mathematics Curriculum Guide*:

- The Western and Northern Canadian Protocol (WNCP) for Collaboration in Education: *The Common Curriculum Framework for K-9 Mathematics*, May 2006. Reproduced (and/or adapted) by permission. All rights reserved.

- Alberta Education (Department of Education)

- Newfoundland and Labrador Department of Education

- Prince Edward Island Department of Education

- The Elementary Mathematics Curriculum Development Advisory Committee

- The Grade 6 Curriculum Development Team:

    - Kathleen Cooper, School District 2
    - Ron Heatherington, School District 17
    - Brenda Johnston, School District 18
    - Josh Little, School District 8
    - Nancy McAlinden, School District 14
    - Susan Vienneau, School District 16

- Cathy Martin, Learning Specialist, K-8 Mathematics and Science, NB Department of Education

- Mathematics Learning Specialists, Numeracy Leads, and mathematics teachers of New Brunswick who provided invaluable input and feedback throughout the development and implementation of this document.

2010
Department of Education and Early Childhood Development
Curriculum K-12 Branch

Table of Contents

BACKGROUND AND RATIONALE

Mathematics curriculum is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society.

It is essential the mathematics curriculum reflects current research in mathematics instruction. To achieve this goal, the Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in New Brunswick. The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the NCTM.

There is an emphasis in the New Brunswick curriculum on particular key concepts at each grade which will result in greater depth of understanding and ultimately stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

The intent of this document is to clearly communicate high expectations for students in mathematics education to all education partners. Because of the emphasis placed on key concepts at each grade level, time needs to be taken to ensure mastery of these concepts. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM Principles and Standards, 2000).

BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

The New Brunswick Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

mathematics learning is an active and constructive process;

learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates;

learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort; and

learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

GOALS FOR MATHEMATICALLY LITERATE STUDENTS

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity.

OPPORTUNITIES FOR SUCCESS

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices. Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals. Striving toward success, and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

DIVERSE CULTURAL PERSPECTIVES

Students attend schools in a variety of settings including urban, rural and isolated communities. Teachers need to understand the diversity of cultures and experiences of all students.

Aboriginal students often have a whole-world view of the environment in which they live and learn best in a holistic way. This means that students look for connections in learning and learn best when mathematics is contextualized and not taught as discrete components. Aboriginal students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized. It is important to note that these general instructional strategies may not apply to all students.

A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students. The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks, 1993).

ADAPTING TO THE NEEDS OF ALL LEARNERS

Teachers must adapt instruction to accommodate differences in student development as they enter school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom. The reality of individual student differences must not be ignored when making instructional decisions.

As well, teachers must understand and design instruction to accommodate differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

CONNECTIONS ACROSS THE CURRICULUM

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, and physical education.

ASSESSMENT

Ongoing, interactive assessment (*formative assessment*) is essential to effective teaching and learning. Research has shown that formative assessment practices produce significant and often substantial learning gains, close achievement gaps and build students' ability to learn new skills (Black & William, 1998, OECD, 2006). Student involvement in assessment promotes learning. Interactive assessment, and encouraging self-assessment, allows students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes:

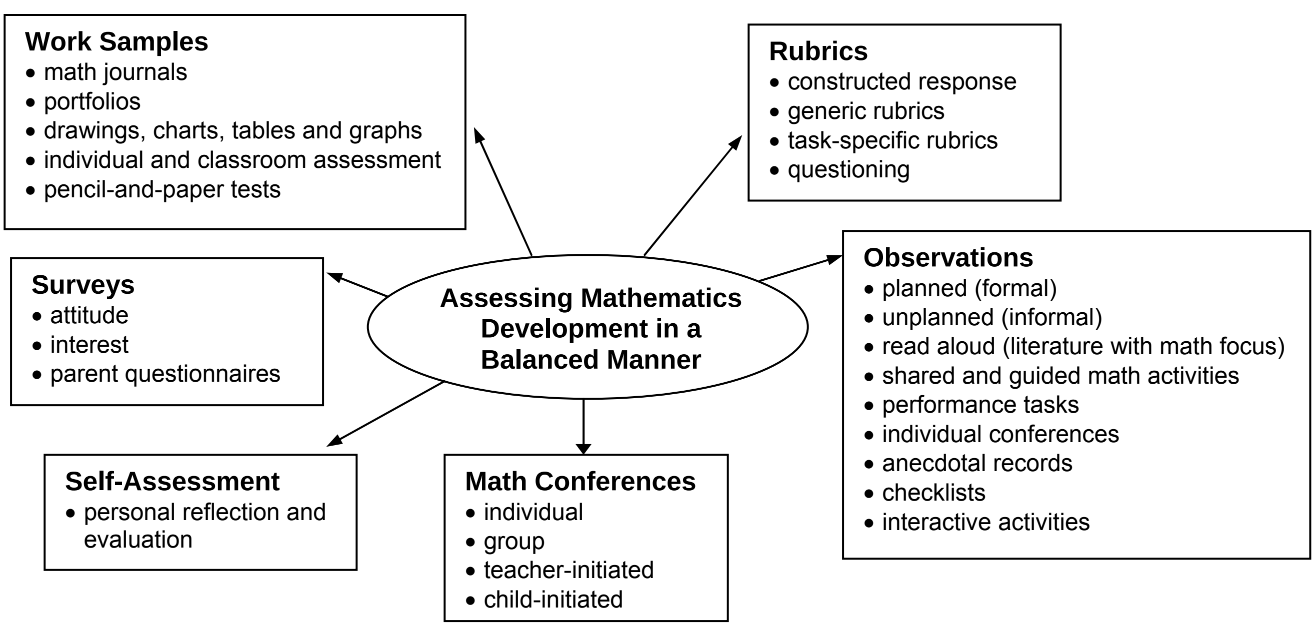
- providing clear goals, targets and learning outcomes
- using exemplars, rubrics and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning (Davies, 2000).

Formative assessment practices act as the scaffolding for learning which, only then, can be measured through summative assessment. *Summative assessment,* or assessment *of* learning, tracks student progress, informs instructional programming and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning and produce achievement gains.

Student assessment should:

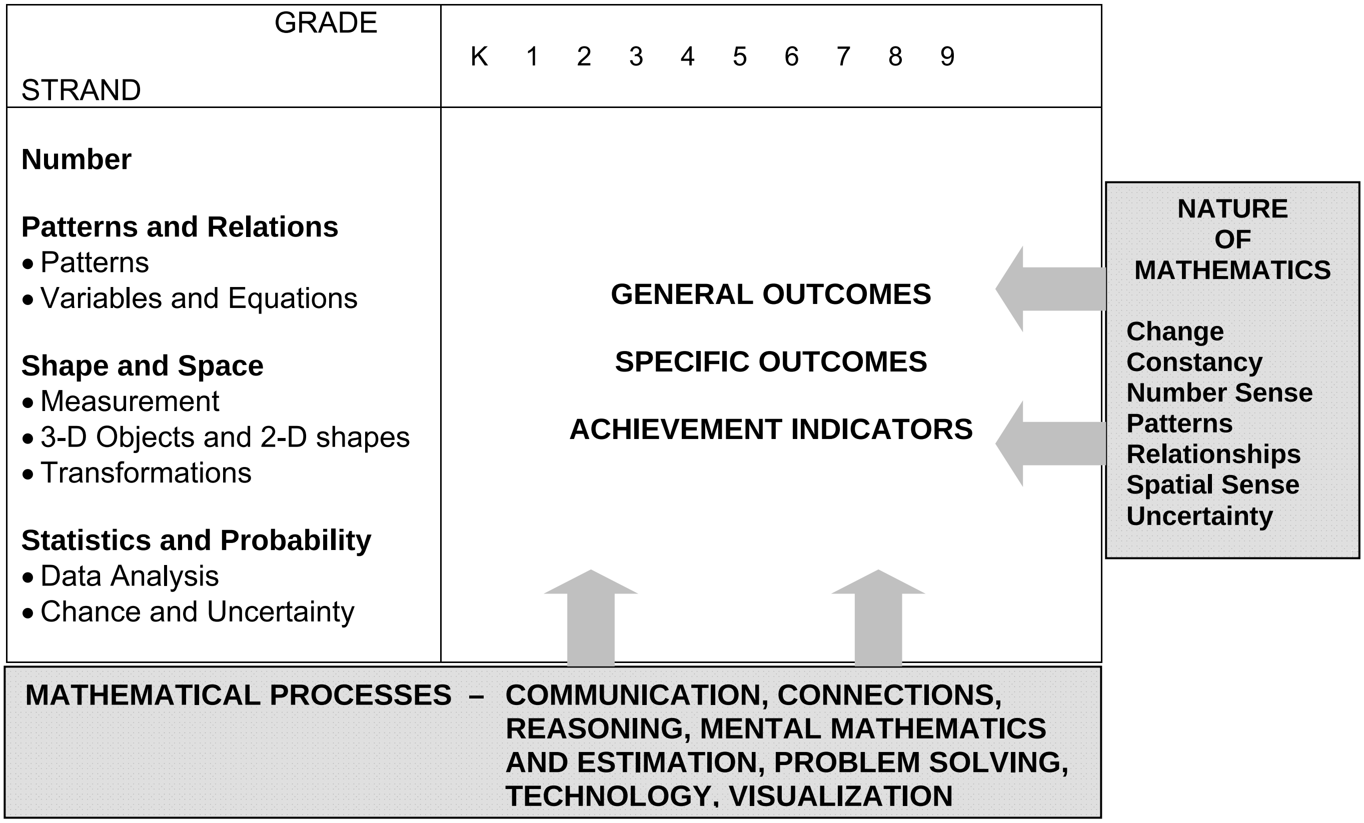
- align with curriculum outcomes
- use clear and helpful criteria
- promote student involvement in learning mathematics during and after the assessment experience
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction

(adapted from: NCTM, Mathematics Assessment: A practical handbook, 2001, p.22



CONCEPTUAL FRAMEWORK FOR K – 9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



INSTRUCTIONAL FOCUS

The New Brunswick Curriculum is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.
- There is a greater emphasis on mastery of specific curriculum outcomes.

The mathematics curriculum describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed. The components are not meant to stand alone. Activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between strands.

MATHEMATICAL PROCESSES

Students are expected to:

- communicate in order to learn and express their understanding of mathematics (Communications: C)
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines (Connections: CN)
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation: ME)
- develop and apply new mathematical knowledge through problem solving (Problem Solving: PS)
- develop mathematical reasoning (Reasoning: R)
- select and use technologies as tools for learning and solving problems (Technology: T)
- develop visualization skills to assist in processing information, making connections and solving problems (Visualization: V).

The New Brunswick Curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"*Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*" (Caine and Caine, 1991, p. 5).

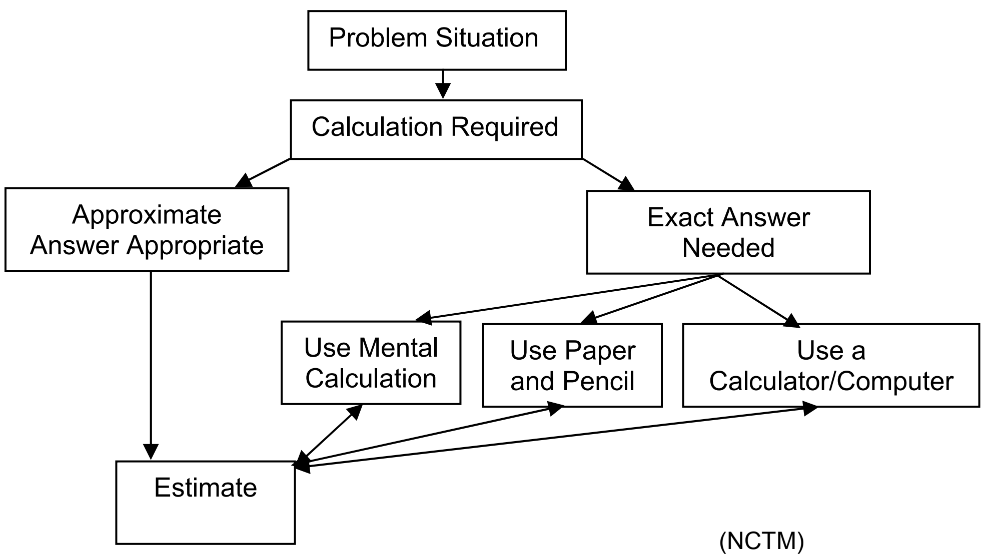
Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mental Mathematics and Estimation [ME]
Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005).
Students proficient with mental mathematics "*become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving*" (Rubenstein, 2001). Mental mathematics "*provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers*" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision making process as described below.



(NCTM)

Problem Solving [PS]
Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "*How would you...?*" or "*How could you...?*" the problem-solving approach is being modeled. Students develop their own problem-solving strategies by being open to listening, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

Technology [T]
Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.
Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K–3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]
Visualization "*involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world*" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: **change**, **constancy**, **number sense**, **relationships**, **patterns**, **spatial sense** and **uncertainty**.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12 … can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

STRUCTURE

STRANDS

The learning outcomes in the New Brunswick Curriculum are organized into four strands across the grades, K–9. Strands are further subdivided into sub-strands which are the general curriculum outcomes.

OUTCOMES AND ACHIEVEMENT INDICATORS

The New Brunswick Curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes and achievement indicators.

General Curriculum Outcomes (GCO) are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

Specific Curriculum Outcomes (SCO) are statements that identify specific concepts and related skills underpinned by the understanding and knowledge attained by students as required for a given grade.

Achievement Indicators are one example of a representative list of the depth, breadth and expectations for the outcome. Achievement indicators are pedagogy and context free.

Strand	General Curriculum Outcome (GCO)
Number (N)	**Number:** Develop number sense
Patterns and Relations (PR)	**Patterns**: Use patterns to describe the world and solve problems
	Variables and Equations: Represent algebraic expressions in multiple ways
Shape and Space (SS)	**Measurement**: Use direct and indirect measure to solve problems
	3-D Objects and 2-D Shapes: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them
	Transformations: Describe and analyze position and motion of objects and shapes
Statistics and Probability (SP)	**Data Analysis**: Collect, display and analyze data to solve problems
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty

CURRICULUM FORMAT

This guide presents the mathematics curriculum by grade level so that a teacher may readily view the scope of the outcomes which students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings at a particular grade level are part of a bigger picture of concept and skill development.

As indicated earlier, the order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The specific curriculum outcomes (SCOs) are presented on individual four-page spreads as illustrated below.

GCO:
SCO: (specific curriculum outcome and mathematical processes)
Key for mathematical processes

Scope and Sequence
Current Grade

Elaboration
Guiding Questions

(Describes the "big ideas" and what the students should learn this year in regards to this concept.)

Page 1

GCO:
SCO:

Achievement Indicators

Guiding Questions

(Describes what could be observed to determine whether students have met the specific outcome.)

Page 2

GCO:
SCO:

Planning for Instruction
Guiding Questions

Choosing Instructional Strategies
(Lists general strategies to assist in teaching this outcome.)

Suggested Activities
(Lists possible specific activities to assist students in learning this concept.)

Possible Models

Page 3

GCO:
SCO:

Assessment Strategies
Guiding Questions

(Overview of assessment)

Whole Class/Group/Individual Assessment
(Lists sample assessment tasks.)

Follow-up on Assessment
Guiding Questions

Page 4

SCO: **N1: Demonstrate an understanding of place value for numbers:**
- **greater than one million**
- **less than one thousandth.**

[C, CN, R, T]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
N1 Represent and describe whole numbers to 1 000 000. **N8** Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.	**N1** Demonstrate an understanding of place value for numbers: • greater than one million • less than one thousandth.	**N2** Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students will extend their knowledge of numbers to one million by discovering patterns that go beyond the millions to the billions, trillions, etc. Students should understand that the place value system follows a pattern such that:
- each position represents ten times as much as the position to its right;
- each position represents one tenth as much as the position to its left;
- positions are grouped in threes for purposes of reading numbers;
- when writing numbers, spaces (not commas) are used to show the positions with the exception of 4-digit numbers (e.g., 5640).

All students should be aware that numbers extend to the left up to infinity, and to the right into the ten thousandths, hundred thousandths and millionths places, and so on.
Students should have many opportunities to:
- read numbers several ways: for example, 6732.14 could be read as six thousand, seven hundred thirty-two and fourteen hundredths or sixty-seven hundred, thirty-two and fourteen hundredths;
- read numbers greater than a million: 2 456 870 346 is read two billion, four hundred fifty-six million, eight hundred seventy thousand, three hundred forty-six ("and" is used for decimal numbers);
- record numbers: for example, ask students to write twelve million, one hundred thousand in **standard form** (12 100 000) and **decimal notation** (12.1 million) (scientific notation is included in later grades);
- establish **personal referents** to develop a sense of larger numbers (e.g., local arena holds 500 people, population of their town is 10 000, a school/class collection of over a million small objects).

Through these experiences, students will develop flexibility in identifying and representing numbers beyond 1 000 000. It is also important for students to gain an understanding of the relative size (magnitude) of numbers through real life contexts that are personally meaningful (e.g., computer memory size, professional athletes' salaries, Internet search responses, populations, or the microscopic world).

Students also need to know that the place value system extends to the right as well and that there are numbers smaller than 0.001.

SCO: **N1: Demonstrate an understanding of place value for numbers:**
- **greater than one million**
- **less than one thousandth.**

[C, CN, R, T]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Explain how the pattern of the place value system, e.g., the repetition of ones, tens and hundreds, makes it possible to read and write numerals for numbers of any magnitude.
- Provide examples of where large numbers and small decimals are used, e.g., media, science, medicine, technology.

SCO: **N1: Demonstrate an understanding of place value for numbers:**
- **greater than one million**
- **less than one thousandth.**

[C, CN, R, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Ask students to find various representations for multi-digit and decimal numbers in newspapers and magazines. Encourage discussion on the need for accuracy in reporting these numbers and the appropriate use of rounded numbers.
- Present a metre stick as a number line from zero to one billion. Ask students where one million, half a billion, one hundred million, etc., would be on this number line.
- Write decimals using place value language and expanded notation to help explain equivalence of decimals.

0.2	= 2 tenths	Since adding zeros have
0.20	= 2 tenths + 0 hundredths	no effect, 0.2 must equal
0.200	2 tenths + 0 hundredths + 0 thousandths	0.20 and 0.200

- Ensure that proper vocabulary is used when reading all numbers. Provide opportunities for students to read decimals in context. Saying decimals correctly will help students make the connection between decimals and fractions 5.0072 should be read as "five **and** seventy-two ten thousandths" not "five point zero, zero, seven, two". Explore the relationship between decimals and related fractions.
- Include contexts that lend themselves to using large numbers such as astronomical data and demographic data. Contexts that lend themselves to decimal thousandths include sports data and metric measurements. An interesting activity involving decimals might require students to complete a chart such as: in 0.1 years, I could...; in 0.01 years, I could...; in 0.001 years, I could...

Suggested Activities

- Create an "A-B-C" book that includes real-world examples of very large numbers and very small decimals (e.g., population of Mexico City; length of an ant's antenna in centimetres).
- Prepare and shuffle 5 sets of number cards (0-9 for each set). Have the student select nine cards and ask him/her to arrange them to make the greatest possible and least possible whole number. Have the student read each of the numbers. Consider extending the activity by asking the student to determine:
  - how many different whole numbers could be made using the nine digits selected;
  - the number of $1000 bills one would get if the greatest and least numbers represented money amounts. This could be extended to explore the number of tens, hundreds, etc. in the number.
- Discuss words people use for large numbers that do not exist (e.g., gazillion, bazillion). Students can explore numbers greater than trillion and look for patterns in the names.
- Ask students to determine the number of whole numbers between 2.03 million and 2.35 million.
- Ask the student to find a value between 0.0001 and 0.00016.
- Present this library information to students: Metropolitan Toronto Library 3 068 078 books; Bibliothèque de Montreal 2 911 764 books; North York Public Library 2 431 655 books. Ask the students to rewrite the numbers in a format such as □ .□million or □ .□□million books. Then ask them to make comparison statements about the number of books.
- Construct a cubic metre and explore the quantity of centimetre cubes will fit in the large cube.

Possible Models: number lines, base ten blocks, thousands grid, decimal squares, Cuisenaire® rods, metre sticks

SCO: **N1: Demonstrate an understanding of place value for numbers:**
- **greater than one million**
- **less than one thousandth.**

[C, CN, R, T]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have students explain at least three things they know about a number with 10 digits.
- Ask a student to describe when 1 000 000 000 of something might be a big amount? A small amount?
- Have students generate a number with 7 – 10 digits. Have them find classmates with numbers that are similar (place value). Once they have found a group they belong to, have them order their numbers from least to greatest. Then have the class order the numbers from least to greatest. Have each student read their number. (This activity can be done in silence so students have to really look at the other numbers). This activity can be done using decimal numbers as well.
- Ask the student to express 0.00674 in at least three different ways.
- Have students write the standard form of numbers that include decimals and/or whole numbers.
  - two hundred and thirty-seven thousandths
  - two hundred thirty-seven thousandths
- Ask the student to describe how the bolded digits in the following two numbers are the same and how they are different.

  5**4**6 397 305 3**4**8 167 903 927

  Extend the activity to decimals:

  0.00**7**0 0.000**7**
- Ask the student to write a report on what he/she has learned about decimals and what questions he/she may now have concerning the topic.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N2: Solve problems involving large numbers, using technology.**
[ME, PS, T]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
N2 Use estimation strategies including: front-end rounding; compensation; compatible numbers in problem-solving contexts.	**N2** Solve problems involving large numbers, using technology.	

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students should continue to use the four operations to solve problems with large numbers. They should also have the opportunity to create problems with large numbers for others to solve. Problem solving situations should be embedded in a meaningful context as often as possible. Provide students with opportunities to solve unfamiliar types of problems and encourage students to persevere. It is important that students communicate their thinking processes about the problem and discuss their strategies with other students.

Technology such as calculators and computers are often useful tools and time-saving devices when solving problems with large numbers. However, students need to determine when the use of these tools is appropriate and when mental math or other strategies are more appropriate (e.g., $12 000 000 is won in the lottery and it is shared with 3 winners. How much does each person receive?). As shown in the chart on page 8 of this document, students need to consider the context of the question and the numbers involved when deciding which approach is most appropriate. Mental math should be considered as a potential approach for solving calculations before technology is used.

Students should also be encouraged to **estimate** answers to test for reasonableness either before or after the calculation. Students should not assume an answer determined with technology is automatically correct. Having students determine the reasonableness of an answer and explaining their thinking is a powerful way to assess understanding and learning.

There are many interesting sources of large numbers, both on the Internet and in reference books. Possible examples that could be discussed are world populations, quantity of data electronic devices can hold (e.g., gigabyte), salaries, and astronomy.

SCO: **N2: Solve problems involving large numbers, using technology.**
[ME, PS, T]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify which operation is necessary to solve a given problem and solve it.
- Determine the reasonableness of an answer.
- Estimate the solution and solve a given problem.

SCO: **N2: Solve problems involving large numbers, using technology.**
[ME, PS, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students research populations of cities and/or provinces in Canada and cities and/or countries of the world. Using this information, students can estimate differences, compare populations and draw conclusions about Canada compared to the world.
- Investigate the concept of “billions”. Although these are rarely found in students' experiences, numbers of this magnitude relate to national debt, personal fortunes, populations, pieces of trivia (e.g., "How long is a billion millimetres?").
- Use information from the above sources to create problems for their classmates to solve. Students are asked to estimate then use technology to check answers.
- Include print resources such as the Canadian Global Almanac, the Guiness Book of World Records, World Atlas and the Top Ten of Everything. Use children literature such as: “If the World Were a Village” by David J. Smith, or “On Beyond a Million” by David Schwartz, to provide a context for large numbers to create and solve problems.
- Do Internet searches for data relating to any topic of student interest, such as sports, money or populations. Useful Internet sources include: Statistics Canada, Canada Population Clock, World Population Clock, *Top Ten of Everything*. Students can also explore the number of responses (“hits”) they get when searching for information on the Internet.

Suggested Activities

- Provide the student with appropriate data and ask him/her to determine how much farther away Jupiter is from Earth than from Mars.
- Ask the students to create a variety of “outlandish” problems involving lengths. For example:
  - How many toothbrushes are required to make a line that is 2 km long?
  - How many pennies must be lined up to make a kilometre?
- Have students create problems based on information provided, such as the following:

	Population 2009
World	6 767 805 208
China	1 338 612 968
United States	307 212 123
Japan	127 078 679
Germany	82 509 367
Canada	32 440 970

Possible Models: calculators, computers

SCO: **N2: Solve problems involving large numbers, using technology.**
[ME, PS, T]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have students look at the area of countries in the world and draw conclusions and comparisons with Canada. Students can present their findings in the form of a project using technology, graphs, and/or illustrations.
- Provide students with information regarding the populations of some of the capital cities of Canada.

City	Population
Fredericton	50,535
Charlottetown	32,174
Halifax	403,200
St. John's	192,300
Toronto	5,741,400
Winnipeg	753,600
Edmonton	1,176,300
Ottawa	1,239,100

  a. If we combined the population of all of the cities in the chart, except Toronto, would it be more or less than the population of Toronto?
  b. About how many more people live in Toronto than Ottawa?
  c. Marc added the population for the capitals of the four Atlantic provinces and said the answer is more than the population of Winnipeg. Is this reasonable? Explain.

- Have students choose two types of careers in entertainment (professional athletes, actors, singers). Have them research the top 5 salaries in each career. Have them generate questions for others to solve and include an answer key. This can be presented in the form of a project.
- Ask students to describe a situation when it would be more appropriate to use a calculator to solve a problem. Have them explain why they would use a calculator rather than pencil and paper to get the answer.
- Provide students with problems involving large numbers and have them estimate the solution and solve the problems. Have students explain their approach and the operations that were used.
- Give students a virtual amount of money to spend (i.e., spend a million dollars or a billion) and have them research from the internet, catalogues etc. items they would buy to make the amount. Students could create a poster or other display to communicate their spending.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N3: Demonstrate an understanding of factors and multiples by:** • **determining multiples and factors of numbers less than 100** • **identifying prime and composite numbers** • **solving problems involving multiples.** [PS, R, V]
[C] Communication **[T]** Technology **[PS]** Problem Solving **[V]** Visualization **[CN]** Connections **[R]** Reasoning **[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
N3 Apply mental mathematics strategies and number properties, such as: skip counting from a known fact; using doubling or halving; using patterns in the 9s facts; using repeated doubling or halving to determine answers for basic multiplication facts to 81 and related division facts.	**N3** Demonstrate an understanding of factors and multiples by: • determining multiples and factors of numbers less than 100 • identifying prime and composite numbers • solving problems involving multiples.	**N1** Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0.

ELABORATION

Guiding Questions:

• *What do I want my students to learn?*
• *What do I want my students to understand and be able to do?*

Multiples of a whole number are the products of that number and any other whole number. To find the first four multiples of 3, multiply 3 by 1, 2, 3, and 4 to get the multiples 3, 6, 9, 12. Multiples of a number can also be found by skip counting by that number.

Factors are numbers that are multiplied to get a **product** (3 and 4 are factors of 12). The factors for a number can be found by dividing the number by smaller numbers and looking to see if there is a remainder of zero. At this point students should also recognize that:
the factors of a number are never greater than the number;
the greatest factor is always the number itself; the least factor is one;
the second factor is always half the number or less (unless the number is prime);
the multiple of a number always has that number as a factor.
To help students understand the meanings for the terms "factor" and "multiple" students could explore these concepts and write their own definition (e.g., *factor* × *factor* = *multiple*).

A **prime** number is defined as a number which has only 2 factors: 1 and itself (e.g., 29 only has factors of 1 and 29 and is therefore prime). Students should recognize that the concept of prime numbers applies only to whole numbers. A "**composite**" number is a number with more than two factors and includes all non-prime numbers other than one and zero (e.g., 9 has factors of 1, 3, 9). It is important for students to realize that <u>0 and 1 are not classified as prime or composite numbers</u>**.** The number "one" has only one factor (itself). Zero is not prime because it has an infinite number of divisors and it is not composite because it cannot be written as a product of two factors that does not include 0.

Although students should have strategies for determining whether or not a number is prime, it is not essential for them to be able to quickly recognize prime numbers. However, students should be able to readily identify even numbers (other than 2) as non-primes (composites) as they will have a minimum of three factors: 1, 2 and the number itself.

Students should be encouraged to accurately use language such as multiple, factor, prime and composite. As well, encourage students to explore numbers and become familiar with their composition.

SCO: **N3: Demonstrate an understanding of factors and multiples by:**
- **determining multiples and factors of numbers less than 100**
- **identifying prime and composite numbers**
- **solving problems involving multiples.**

[PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify multiples for a given number and explain the strategy used to identify them.
- Determine all the whole number factors of a given number using arrays.
- Identify the factors for a given number and explain the strategy used, e.g., concrete or visual representations, repeated division by prime numbers or factor trees.
- Provide an example of a prime number and explain why it is a prime number.
- Provide an example of a composite number and explain why it is a composite number.
- Sort a given set of numbers as prime and composite.
- Solve a given problem involving factors or multiples.
- Explain why 0 and 1 are neither prime nor composite.

SCO: **N3: Demonstrate an understanding of factors and multiples by:**
- **determining multiples and factors of numbers less than 100**
- **identifying prime and composite numbers**
- **solving problems involving multiples.**

[PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

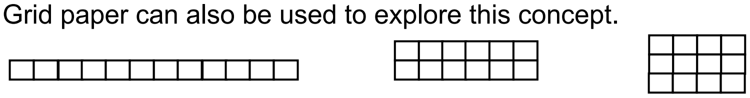
Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

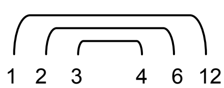
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

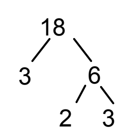
- Have students determine the factors of a number by arranging square tiles into as many different arrays (rectangles) as possible. Record the unit length and width of each rectangle. For example, if 12 tiles were used, the rectangles would be 1 by 12, 2 by 6, and 3 by 4. These are the factor pairs for 12. Have students record their rectangles/factor partners on grid paper. Students should discover that some numbers only have one rectangle. This is an effective approach to introducing prime numbers. Grid paper can also be used to explore this concept.



- Have students investigate other numbers to find their factor pairs. Students may use organized lists to determine factors (i.e., begin with 1 and the number itself, then 2 or the next possible factor and its factor partner, etc.)



- Have students factor odd composite numbers (e.g., 33, 39). Students sometimes mistake these for prime as they do not readily see how they are factored.
- Have students skip count or number lines or use other models such as same colour Cuisenaire® rods or equal lengths of connected base ten unit cubes to find multiples of a number.
- Explore other strategies such as factor trees to determine prime and composite numbers.



Suggested Activities

- Explore the sieve of Eratosthenes to identify the prime numbers to 100. On a hundreds chart, have the students begin by circling the first prime number, 2, and then cross out all the multiples of 2 (composite numbers). Circle the next prime number, 3, and cross out all of its multiples. Students then proceed to the next number that has not been crossed off and repeat the procedure. At the end of the process the circled numerals will be the prime numbers up to 100. Discuss any patterns they notice.
- Have students express even numbers greater than 2 in terms of sums of prime numbers. (Sample answers may include 4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, ..., 48 = 43 + 5, 50 = 47 + 3, ...). Explore this idea further by asking if every even number greater than 2 can be written as the sum of 2 primes (Goldbach's Conjecture).
- Ask students to name numbers with a given amount of factors (e.g., numbers with 6 factors: 12, 18, 20, etc.).
- Have students use the constant function on their calculators to explore multiples of a number. They may also use calculators to systematically test for factors of a number: ÷ 1, ÷ 2, ÷ 3, ÷ 4, etc.

Possible Models: grid paper, coloured tiles, hundreds chart, geoboards, Cuisenaire® rods, metre sticks, base ten blocks

SCO: **N3: Demonstrate an understanding of factors and multiples by:**
- **determining multiples and factors of numbers less than 100**
- **identifying prime and composite numbers**
- **solving problems involving multiples.**

[PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask students to express 36 as the product of two factors in as many ways as possible.
- Have small groups of students find the number less than 50 (or 100) that has the most factors. Ensure students can explain their process and justify their answer.
- Have students show all the factors of 48 by drawing or colouring arrays on square grid paper.
- Have students solve problems involving factors and multiples, such as the following:
  - Mr. Reeves has 24 students in his class. How many different size groups of students can he make so that all groups are the same size? (1, 2, 3, 4, 6, 8, 12, 24)
- Ask students if it is possible to list all of the multiples of 12? Explain their reasoning.
- Have students list all of the factors of 8 and the first ten multiples of 8.
- Ask students to explain, without dividing, why 2 cannot be a factor of 47.
- Ask students to identify a number with 5 factors.
- Ask students to find 3 pairs of prime numbers that differ by two (e.g., 5 and 7).
- Ask: Why is it easy to know that certain large numbers (e.g., 4 283 495) are not prime, even without factoring them?
- Tell students that the numbers 2 and 3 are consecutive numbers, both of which are prime numbers. Ask: Why can there be no other examples of consecutive prime numbers?
- Have students use a computer or calculator to help them determine the prime numbers up to 100. Ask them to prepare a report describing as many features of their list as they can.
- Have students draw diagrams (such as rectangles or factor rainbows) to show why a given number is or is not prime (e.g., 10, 17, 27).

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N4: Relate improper fractions to mixed numbers.** [CN, ME, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
N7 Demonstrate an understanding of fractions by using concrete and pictorial representations to: create sets of equivalent fractions; compare fractions with like and unlike denominators.	**N4** Relate improper fractions to mixed numbers.	**N5** Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

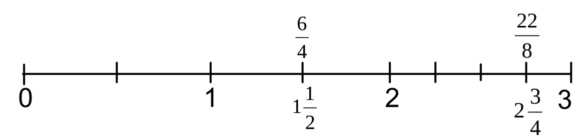
ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In Grade 6, students extend their understanding of fractions to learn that an **improper fraction** represents a fraction greater than one. Through the use of models, students should discover that fractions with the numerator greater than their denominator are greater than one (e.g., $\frac{5}{3}$, $\frac{6}{2}$, $\frac{7}{6}$). It is important for students to understand that an improper fraction can also be expressed as a **mixed number** which is a whole number and a **proper fraction** (e.g., $1\frac{1}{4}$).

Students should fluently move between the mixed number and improper fraction formats of a number. Rather than only applying a rule to move from one format to the other, students should be encouraged to focus on the meaning. For example, since $\frac{14}{3}$ is 14 thirds and it takes 3 thirds to make 1 whole, 12 thirds would equal 4 wholes, so $\frac{14}{3}$ represents 4 wholes and another 2 thirds of another whole or $4\frac{2}{3}$. Often it is easier for students to grasp the magnitude of mixed numbers rather than improper fractions. For example, a student may know that $4\frac{1}{3}$ is a bit more than 4, may not have a good sense of the size of $\frac{13}{3}$.
Students should be able to place mixed numbers and improper fractions on a number line easily when they have **benchmarks** to use such as: closer to zero, close to one half, closer to one, etc. Having these benchmarks helps students visualize the placement and order of these fractions. The concept of equivalent fractions that students learned in Grade 5 will also be helpful in developing additional benchmarks.



It is important that students have an opportunity to explore through a problem solving context and the use of a variety of models, that fractions are connected to multiplication and division. Students should discover that dividing the numerator by the denominator is a procedure that can be used to change an improper fraction to a mixed number. It would be inappropriate just to tell students to divide the denominator into the numerator to change an improper fraction to a mixed number before they develop the conceptual understanding for this.

SCO: **N4: Relate improper fractions to mixed numbers.**
[CN, ME, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Demonstrate using models that a given improper fraction represents a number greater than 1.
- Express improper fractions as mixed numbers.
- Express mixed numbers as improper fractions.
- Place a given set of fractions, including mixed numbers and improper fractions, on a number line and explain strategies used to determine position.

SCO: **N4: Relate improper fractions to mixed numbers.**
[CN, ME, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

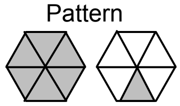
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

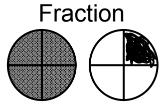
- Explore improper fractions and mixed numbers in a variety of ways and use a variety of different models. Some examples are:

Pattern



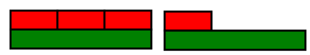
$1\frac{1}{6}=\frac{7}{6}$

Fraction



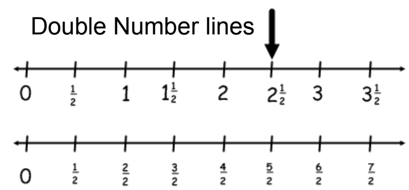
$1\frac{1}{4}=\frac{5}{4}$

Cuisenaire® rods



$1\frac{1}{3}=\frac{4}{3}$

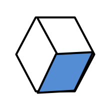
Double Number lines



- Use pattern blocks and have students build and count fractional parts and continue beyond a whole: $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}$, etc. Ask them to show another way to represent the improper fractions (e.g., $\frac{5}{3} = 1\frac{2}{3}$). Gradually transition to doing this activity without the pattern blocks (or other models).
- Provide students with frequent opportunities to use number lines (including double number lines) to explore the placement of mixed numbers and improper fractions. Ensure students are able to explain their strategy focusing on the use of benchmarks.
- Ask students to visualize (picture) fractions based on their experiences with various models. They should be able to draw a variety of representations for the same fraction (e.g., $\frac{7}{3}$).

Suggested Activities

- Ask the student to model mixed numbers and improper fractions in various ways (e.g., $1\frac{3}{4} = \frac{7}{4}$).
- Have students determine what fraction the blue rhombus represents if the hexagon is one whole. Using pattern blocks, have students explain what is another name for $\frac{14}{3}$.



- Have students solve problems such as: Jamir has 15 quarters in his pocket. How many whole dollars does he have?
- Create a set of equivalent mixed number and improper fraction cards and distribute a card to each student. Students need to find their equivalent partner. Then have students line up by pairs in ascending order (a temporary number line on the floor might be helpful for students). This activity should be done after students have had an opportunity to develop their understanding with models.
- Have students model $\frac{9}{4}$ and tell how many groups of 4 are in 9. For example;

1st group of 4 2nd group of 4 $\frac{1}{4}$ of a group of 4

Possible Models: fraction circles, pattern blocks, Cuisenaire® rods, double number lines, colour tiles, fraction pieces, egg cartons

SCO: **N4: Relate improper fractions to mixed numbers.**
[CN, ME, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have students explain how they know that $\frac{5}{4}$ must be more than one whole.
- Ask students: If 14 people at a party each want $\frac{1}{3}$ of a pizza, how many pizzas would be needed?
- Ask students to use coloured tiles to show why $3\frac{1}{3} = \frac{10}{3}$. Observe whether or not they make wholes of 3 (or 6 or 9...) squares.
- Provide students with several mixed numbers and improper fractions that are equivalent (e.g., $2\frac{3}{4} = \frac{11}{4}$). Ask them to show if the numbers are equal and to explain their thinking concretely, pictorially and symbolically.
- Ask students to write as many improper fractions as they can with the numbers: 3, 6, 7, and 8. Have them to represent one of the improper fractions using a model or a picture.
- Have students explain a situation when it would be a good idea to express an improper fraction as a mixed number.
- Write and model a mixed number, with the same denominator, that is greater than $\frac{3}{3}$, but less than $\frac{6}{3}$.
- Provide students with several mixed numbers and improper fractions. Have students place the numbers on an open number line to demonstrate their relative magnitude.

  $2\frac{1}{3}$ $\frac{7}{4}$ $\frac{5}{3}$ $2\frac{3}{4}$ $1\frac{4}{5}$

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N5: Demonstrate an understanding of ratio, concretely, pictorially and symbolically.** [C, CN, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
N7 Demonstrate an understanding of fractions by using concrete and pictorial representations to: create sets of equivalent fractions; compare fractions with like and unlike denominators.	**N5** Demonstrate an understanding of ratio, concretely, pictorially and symbolically.	**N3** Solve problems involving percents from 1% to 100%.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Ratios and fractions are both **comparisons**. A ratio is a way to represent comparisons of two numbers or quantities. It can be used to compare part-to-part in any order or part-to-whole. Ratios may be expressed in words, in number form with a colon between the two numbers, or as a fraction (if it represents part-to-whole). For example, the ratio of 2 boys compared to 5 girls can be expressed as “two to five”, 2 to 5, or 2:5. If the ratio is comparing a part of a set to the whole set, then fractions can also be used (e.g., the number of boys to the whole group would be $\frac{2}{7}$).

The items and the order in which they are being compared must always be stated. For example:

Faces to hearts is 4:1
Hearts to faces is 1:4
Faces to all is 4:5
All to hearts is 5:1

Ratios may be generated in geometric, numerical, and measurement situations. Some examples:
Geometric situations:
- the ratio of the number of sides in a hexagon to the number of sides in a square (6:4);
- the ratio of the number of vertices to the number of edges in a rectangular prism (8:12);
- the ratio of the number of vertices in a hexagon to the number of sides (6:6).

Numerical situations:
- the ratio comparing the value of a quarter to that of a dime (25:10);
- the ratio comparing the number of multiples of 2 to the multiples of 4 for numbers from 1 to 100 (2:1 or 50:25).

Measurement situations:
- the ratio of perimeter to side length of a square (4:1);
- the ratio describing the size of scale models or the scale on a map (1:15).

The concept of rate, as a ratio, will be introduced in Grade 8.

Equivalent ratios can be explored in relation to equivalent fractions. This can be accomplished engaging students with problems within a real-world context. Encouraging students to write and solve their own problems will help them construct and consolidate their understanding of equivalent ratios.

SCO: **N5: Demonstrate an understanding of ratio, concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Provide a concrete or pictorial representation for a given ratio.
- Write a ratio from a given concrete or pictorial representation.
- Express a given ratio in multiple forms, such as “three to five”, 3:5, 3 to 5, or $\frac{3}{5}$.
- Identify and describe ratios from real-life contexts and record them symbolically.
- Explain the part/whole and part/part ratios of a set, e.g., for a group of 3 girls and 5 boys, explain the ratios 3:5, 3:8 and 5:8.
- Solve a given problem involving ratio.

SCO: **N5: Demonstrate an understanding of ratio, concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use counters, other simple models, or the students themselves to introduce the concept of ratio as a comparison between two numbers.
  For example, in a group of 3 boys and 2 girls:
  - 3:2 tells the ratio of boys to girls (part-to-part);
  - 3:5 tells the ratio of boys to the total group (part-to-whole);
  - 2:5 tells the ratio of girls to the total group (part-to-whole);
  - 2:3 tells the ratio of girls to boys (part-to-part).

  Students should read "3:2" as "3 to 2" or "3 __ for every 2 ___."
- Explore ratios that occur in everyday situations (e.g., the ratio of water to concentrate to make orange juice is 3:1 or "3 to 1").
- Use children's literature, such as *If You Hopped Like a Frog!* by David Schwartz, to provide a context for students to explore ratio.
- Have students use colour tiles, pattern blocks, linking chains, or other models to represent ratio comparisons.

Suggested Activities

- Provide students with a recipe for lemonade: 4 cups of water, 1 cup of lemon juice, 1 cup of sugar. Have students to write about the various ratios that can be drawn from this data. Ask which juice will be stronger flavoured: 3 cans of water for 1 can of concentrate or 4 cans of water for 2 cans of concentrate?
- Have students poll their classmates to determine what pets they have (or other topics such as eye colour, shoe size, hair colour, etc.). Ask them to write part-to-part and part-to-whole ratio comparisons. Require students to write their ratios in words, using a colon, and in fraction form (for part-to-whole only).
- Have students model two situations that can be described by the ratio 3:4. Specify that the situations must involve a different total number of items.
- Ask the student to find the following body ratios, comparing results with others:
  - wrist size: ankle size
  - wrist size: neck size
  - head height: full height
- Have students show a given ratio pictorially or concretely. For example, to show 4:5 (part/part), one possible solution would be:

  ○ ○ ○ ○ ● ● ● ● ●

  Have students write 3 other possible ratios for the model.
- Ask students to write a number of ratios that relate to sport or other real world situations. For example, compare the number of players on the ice in hockey compared to the number on a soccer field.
- Encourage students to write and solve each other's ratio problems.

Possible Models: colour tiles, pattern blocks, linking cubes, Cuisenaire® rods, two color counters

SCO: **N5: Demonstrate an understanding of ratio, concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask the student to select up to 20 tiles of different colours so that pairs of colours show the following ratios: 4 to 3, 2:1, $\frac{1}{5}$.
- Give students the following information and ask them to write and read ratio comparisons. Have students explain their ratios. Students should be able to express their ratios as fractions, words, and numbers.

  4 cats 3 goldfish 2 hamsters
- Ask students to make a drawing that shows a ratio situation (e.g., for every one pencil, there are three pieces of paper). Ask them to write ratios to describe what the picture shows and describe the different ratios it represents.
- Tell students that the ratio of boys to the total number of students in the class is 13:28. How many girls are in the class?
- Ask students what would the ratio of legs to heads be in a group of bears? of people? of spiders?
- Ask students to explain to why they might describe the ratio below as 4:1 (all to girls) or as 1:4 (girls to all)? Are there other ratios that can be used to describe what is given? (B = boy; G = girl)

  B B B G
- Have students solve problems such as the following.
  - It is evening and Jolyn is out in the yard with her brother. Jolyn is 1 metre tall and casts a shadow that is 3 metres. If Jolyn stands on her brother's shoulders, which are 1.5 m above the ground, how long a shadow will she and her brother cast?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N6: Demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
N9 Relate decimals to fractions and fractions to decimals (to thousandths).	**N6** Demonstrate an understanding of percent, (limited to whole numbers) concretely, pictorially and symbolically.	**N3** Solve problems involving percents from 1% to 100%.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Percent is a part-to-whole ratio that compares a number to 100. "Percent" means "out of 100" or "per 100". Students should understand that percent on its own does not represent a specific quantity. For example, 90% might represent 9 out of 10, 18 out of 20, 45 out of 50, and 90 out of 100. This is the first year for students to explore this concept.

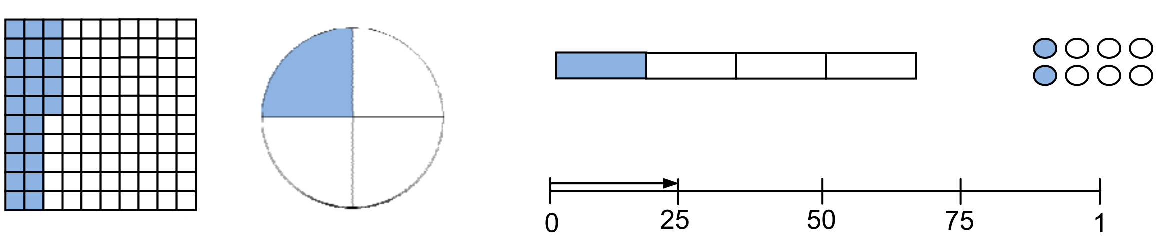
Percent can always be written as a decimal or vice versa. For example, 26% is the same as 0.26, and both mean 26 hundredths or $\frac{26}{100}$.

Students should recognize:
- situations in which percent is commonly used;
- diagrams, showing parts of a set, whole, or measure that represent various percentages (e.g., 2%, 35%);
- the relationship between the percent and corresponding decimals and ratios (e.g., 48%, 0.48, 48:100);
- the percent equivalents for common fractions and ratios such as $\frac{1}{4}$ = 25%, $\frac{1}{2}$ = 50%, and $\frac{3}{4}$ = 75%.

Students do **not** need to compute with percentages or work with percentages greater than 100 in Grade 6.

Number sense for percent should be developed through the use of these basic **benchmarks**:
- 100% is all;
- 50% is half;
- 25% is a quarter; 75% is three quarters;
- 33% is a little less than a third; 67% is a little more than two thirds.

It is important for students to use a variety of representations of percent to help deepen their understanding. For example, 25% can be represented in a variety of ways as shown below.



SCO: **N6: Demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Explain that "percent" means "out of 100."
- Explain that percent is a ratio out of 100.
- Use concrete materials and pictorial representations to illustrate a given percent.
- Record the percent displayed in a given concrete or pictorial representation.
- Express a given percent as a fraction and a decimal.
- Identify and describe percents from real-life contexts, and record them symbolically.
- Solve a given problem involving percents.

SCO: **N6: Demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

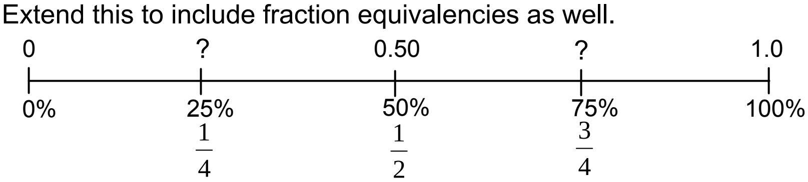
Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

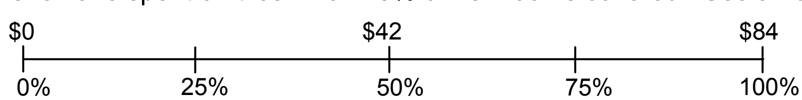
Consider the following strategies when planning lessons:

- Provide students with many opportunities to work with partially shaded hundreds grids, determining the decimal, fraction, ratio, and percent that is shaded.
- Make charts, including symbolic representations, for fractions, decimals, and percents that are equal.
- Use virtual manipulatives that are available on the Internet and interactive whiteboard software.
- Have the students predict percentages, give their prediction strategies, and then check their predictions. For example, ask them to estimate the percentage of:
  - each colour of Bingo chips, if a total of 100 blue, red, and green chips are shown on an overhead for 10 seconds;
  - a hundred grid that is shaded in to make a picture;
  - red counters when 50 two coloured counters are shaken and spilled.
- Use a double number line as a useful tool to model and solve simple percent equivalencies and problems. Extend this to include fraction equivalencies as well.



Suggested Activities

- Ask students to draw a design on a hundred grid and describe the percent that is shaded.
- Have students create a pencil crayon quilt made of patches of various colours. They can describe the approximate or exact percentages of each colour within the patch and then estimate the percent of the total quilt that is each colour.
- Tell students that Jane is covering her floor with tiles. The cost of covering the whole floor is $84. How much will she have spent on tiles when 25% of her floor is covered? Use a number line to help model.



- Have students collect examples of situations from newspapers, flyers, or magazines in which percent is used and have them make a collage for a class display.
- Have students estimate the percent of time students spend each day doing certain activities (e.g., attending school, physical activity, eating, sleeping, etc.).
- Provide students with different size pieces of scrap paper and ask students to tear off about 60% from their piece. Explain their thinking to a partner. Repeat with other percentages.
- Have students estimate and then determine the percentage of pages in a magazine that have advertisements on them.

Possible Models: hundred grids, double number lines, hundredths circles

SCO: **N6: Demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

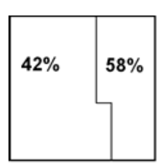
Look back at what you determined as acceptable evidence.

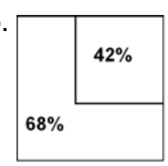
Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask students:
  a. Which is the least? The most? Explain your answer.
     $\frac{1}{20}$ 20% 0.02
  b. Which one doesn't belong? Explain your choice:
     $\frac{3}{4}$ 0.75 0.34 75%
- Ask students what percent of a metre stick is 37 cm?
- Have students examine a set of objects, and describe different ratio and percent equivalents.
- Have students name percents that indicate:
  - almost all of something
  - very little of something
  - a little less than half of something
- Tell students that 60 new floor tiles are being installed in a room. The tiles used must be the following colours: 25% must be blue; 4:10 must be red; 0.20 must be green; the rest are yellow.
- Ask students to describe a situation when 45% can be greater than 90%.
- Have students draw and colour a picture on grid paper to show what the room tiles may look like and explain how they decided how many of each colour needed to be used.
- Ask students what is incorrect about each of the following diagrams:
  Have students justify their answers.

a.


b.


FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N7: Demonstrate an understanding of integers, concretely, pictorially and symbolically.** [C, CN, R, V]
[C] Communication **[T]** Technology **[PS]** Problem Solving **[V]** Visualization **[CN]** Connections **[R]** Reasoning **[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
	N7 Demonstrate an understanding of integers, concretely, pictorially and symbolically.	**N6** Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.

ELABORATION

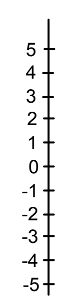
Guiding Questions:
• What do I want my students to learn?
• What do I want my students to understand and be able to do?

Negative numbers have been part of the day-to-day life of students through experiences such as temperatures below zero. Students will be introduced to the set of **integers** which includes positive and negative whole numbers and zero.

The big ideas of integers that students should understand in Grade 6 are:
- each negative integer is the mirror image of a positive integer with respect to the 0 mark therefore the same distance from zero;
- 0 is neither positive nor negative;
- negative integers are all less than any positive integer;
- a positive integer closer to zero is always less than a positive integer farther away from zero (e.g., +3 < +7);
- a negative integer closer to zero is always greater than a negative integer farther away from zero. (e.g., -3 > -7).

Students should be encouraged to read -5 as “negative 5” rather than “minus 5,” to minimize confusion with the operation of subtraction. It is also important for students to recognize that positive integers do not always show the “+” symbol. If no symbol is shown, the integer is positive.

Students will have previously encountered negative integers in several of the above situations, but one of the most common contexts is a thermometer. To build on this informal understanding, it is beneficial to start with a vertical number line which resembles a thermometer, but also include horizontal models. Other useful contexts for considering negative integers are:
- temperatures;
- elevators which go both above and below ground (floors can have positive and negative labels);
- golf scores above and below par;
- money situations involving debits and credits;
- distance above and below sea level.



In prior grades, students compared numbers using the vocabulary of “**greater than**” and “**less than**” and were introduced to the **>** and **<** symbols in Grade 3. In Grade 6, students will be expected to represent comparisons using these symbols.

Addition and subtraction situations involving integers is a Grade 7 outcome, but it can be explored informally with students.

SCO: **N7: Demonstrate an understanding of integers, concretely, pictorially and symbolically.** [C, CN, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Extend a given number line by adding numbers less than zero and explain the pattern on each side of zero.
- Place given integers on a number line and explain how integers are ordered.
- Describe contexts in which integers are used, e.g., on a thermometer.
- Compare two integers, represent their relationship using the symbols <, > and =, and verify using a number line.
- Order given integers in ascending or descending order.

SCO: **N7: Demonstrate an understanding of integers, concretely, pictorially and symbolically.** [C, CN, R, V]

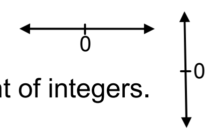
PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies



Consider the following strategies when planning lessons:

- Provide students with open number lines in different orientations to explore the placement of integers.
- Explore examples of situations where negative integers are used in various media.
- Have students divide a sheet of paper into 3 parts with the headings of Negative, Positive, and Zero. As situations arise throughout the teaching of this outcome, have students record the situation under the headings which best describes it. For example, rise in temperature (positive), spending money (negative), freezing point (zero).
- Give each student a card with an integer number on it (ensure that the set of cards includes pairs of integers, such as +7, -7, and a card with zero). Have the person with the “zero” card stand at the front of the classroom in the middle. Have the rest of the students create a “human number line” placing them in order according to the card they were given.
- Use a thermometer (vertical number line) to compare integers and record the comparison symbolically (-8 < 5; 6 > -7; 4 < 9; -3 > -4).

Suggested Activities

- Have 10 students volunteer, to come to the front of the class. They are given an integer unknown to them, on a sticky note, stuck on their backs. The volunteers without talking must rearrange themselves in ascending by moving each other.
- Have the student place a variety of integers at the appropriate places on a number line.
- Have students play the card game “integer war”, using the red cards for negative integers and the black cards as positive integers. Each student flips a card, the student holding the card with the highest value, wins both cards.
- Have students choose 10 cities and research the temperature for a specific date, enter the data into a table from warmest to coldest temperatures. Students may use a vertical number line to facilitate this task.
- Write an integer for each of the following situations:
  - A person walks up 8 flights of stairs.
  - An elevator goes down 7 floors.
  - The temperature falls by 7 degrees.
  - Josh deposits $110 dollars in the bank.
  - The peak of the mountain is 1123 m above sea level.
- Have student investigate **opposite integers** by plotting points such as +5 and -5 on a number line. What do you notice about them? Why do you think number pairs such as -5 and +5 are called opposites?

Possible Models: vertical and horizontal number lines, two colour counters, thermometer, playing cards

SCO: **N7: Demonstrate an understanding of integers, concretely, pictorially and symbolically.** [C, CN, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Provide students with a blank number line. Have them place positive and negative integers on the line and explain their thinking.
  -5 3 0 -2 2 -1 6
- Ask students: How many negative integers are greater than -7?
- Tell students that a number is 12 jumps away from its opposite on a number line. Ask: What is the number?
- Explain why -4 and +4 are closer to each other than -5 and +5.
- Ask students to design a simple game for which positive and negative points might be awarded. Have the students play and keep track of their total scores.
- Ask students why an integer is never an odd number away from its opposite on a number line?
- Have students flip over two playing cards (red cards could represent negative integers and black cards could represent positive integers). Record the comparison symbolically with numbers and the symbols > and <.
- Ask students whether it is true that: (Have them explain their thinking for each.)
  a. a negative number further from zero is less than a negative number that is close to zero;
  b. a negative number is always less than a positive number;
  c. a positive number is always greater than a negative number;
  d. integer opposites cancel each other out when they are combined.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N8: Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).** [C, CN, ME, PS, R, V]
[C] Communication **[T]** Technology **[PS]** Problem Solving **[V]** Visualization **[CN]** Connections **[R]** Reasoning **[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
N5 Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems. **N6** Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.	**N8** Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).	**N2** Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students will have had experience multiplying and dividing whole numbers in previous grades. The emphasis will continue to be on the understanding of these two operations rather than the mastery of one traditional algorithm. As students extend their learning to multiplying and dividing with decimals the use of **estimation** is essential to help students ensure the **reasonableness** of their answer. “When estimating, thinking focuses on the meaning of the numbers and the operations, and not on counting decimal places” (Van de Walle & Lovin, vol. 3, 2006, p. 125).

When considering the multiplication of decimals, students should recognize that, for example, 0.8 of something will be almost that amount, but not quite, and 2.4 multiplied by an amount will be double the amount plus almost another half of it. It is important for students to realize estimation is a useful skill in their lives and regular emphasis on real-life contexts should be provided. On-going practice in computational estimation is a key to developing understanding of number and number operations and increasing mental process skills. Although rounding has often been the only estimation strategy taught, there are others (many of which provide a more accurate answer) that should be part of a student's repertoire such as front-end estimation:

- Multiplication: 6 × 23.4 might be considered to be 6 × 20 (120) plus 6 × 3 (18) plus a little more for an estimate of 140, or 6 × 25 = 150.
- Division: Pencil and paper division involves front-end estimation. To solve $8\overline{)424.53}$ (or 424.53 ÷ 8) students should be able to estimate that 50 × 8 is 400, so the quotient must be a bit more than 50.

Students should be able to place missing decimals in products and quotients using estimation skills and not rely on a rule for “counting” the number of digits without any conceptual understanding.

A connection should be made between multiplication and division. Multiplication can be used to estimate **quotients**. For example, 74.3 divided by 8. Have a student say the multiples of 8 that are closest to 74.3. Write out 8 × 9 = 72 and 8 × 10 = 80. Students should explain how they know the quotient is between 9 and 10. Ensure proper vocabulary when reading all numbers. This will assist students in making the connection between facts (e.g., 4 × 6 is similar to 4 × 0.6; 4 groups of 6 tenths = 24 tenths or 2.4).

Students should have frequent opportunities to solve and create word problems for the purpose of answering real-life questions of personal interest. These opportunities provide students with a chance to practice their computational skills and clarify their mathematical thinking.

SCO: **N8: Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).**
[C, CN, ME, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Place the decimal point in a product using front-end estimation, e.g., for 15.205 m × 4, think 15 m × 4, so the product is greater than 60 m.
- Place the decimal point in a quotient using front-end estimation, e.g., for $25.83 ÷ 4, think $24 ÷ 4, so the quotient is greater than $6.
- Correct errors of decimal point placement in a given product or quotient without using paper and pencil.
- Predict products and quotients of decimals using estimation strategies.
- Solve a given problem that involves multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9.

SCO: **N8: Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).**
[C, CN, ME, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

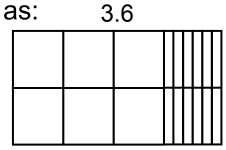
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Ensure students use proper vocabulary related to multiplication (factors, product) and division (divisor, dividend, quotient) that they have learned in previous grades.
- Have students look for benchmark decimals that are easy to multiply and divide. For example, ask the students why someone might estimate 516 × 0.48 by taking half of 500.
- Provide opportunities for students to create and solve missing factor and missing divisor/dividend problems, involving decimals, to support the connection between multiplication and division.
- Use the "area model" both concretely with base ten blocks and pictorially to represent multiplication and division before moving to the symbolic. For example, 2 × 3.6 could be modelled as: Additional examples of the array model can be found in the Grade 5 curriculum.

3.6

2



Suggested Activities

- Provide students with a number sentence that has decimals missing or misplaced in either the answer or the question. For example, 2.34 × 6 = 1404 a decimal is missing in the product. Have students determine where the decimal should be using estimation strategies such as "front-end".
- Have students estimate each of the following and tell which of their estimates is closer and how they know:
  - 3 videos games at $24.30/game OR 5 teen magazines at $8.89/magazine;
  - 9 glasses of fruit smoothies at $2.59/glass OR 4 veggie pitas at $4.69/pita.
- Tell students that it takes about 9 g of cookie dough to make one cookie. Renee checks the label on the package and finds she has 145.6 g of dough. About how many cookies can she make?
- Have students measure side lengths of objects in the classroom to the nearest tenth of a centimetre or hundredth of a metre and then estimate the area of those objects (e.g., side lengths of their desks, their textbooks or the top of tables).
- Have students solve problems which involve dividing the price for a pizza. For example, 4 people sharing a pizza for $14.56. Change the amount of people and the price of the pizza for more problems.
- Tell students that the cashier told Samantha that her total for 3 kg of grapes at $3.39/kg was $11.97. How did Samantha use estimation to know that the cashier had made a mistake?
- Provide real-world problems involving multiplication and division of decimals where the multiplier/divisor are 1-digit whole numbers. For example, Jean works at Pizza Pie for $8.75/hour. Saturday he worked 8 hours. What were his earnings? Sunday he made $93.25, and was paid $9.00 per hour. How many hours did he work?
- Have students figure out how much they need to pay, if they went to the restaurant with three friends and the bill came to $26.88. Students should assume that each person pays their equal share.

Possible Models: base ten blocks, ten frames (combined to make one hundred), number lines, metre sticks, place value chart, money, calculator, area models, grid paper, open array

SCO: **N8: Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).**
[C, CN, ME, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Tell students that you have multiplied a decimal by a whole number and the estimated product is 5.5. What might the two numbers be?
- Ask students to draw or build a model to illustrate 3 × $2.80 and show the answer. Ask students to create a story problem based on the multiplication sentence and share it with a partner to solve.
- Provide a student with a supermarket checkout slip and tell them that it represents a family's weekly groceries. Have the student estimate the total amount spent per day or per month by that family.
- Ask students for an estimate of the total cost of 8 pens at $0.79 each. Ask what estimating strategy he/she used and if there is another way to easily estimate the answer.
- Tell students that the class wants to buy 6 pizzas that cost $11.85 each. How much money will they need? What is the best choice for the correct answer?
  a. 7.11 b. 71.10 c. 711.0
- Have students estimate the mass of each egg in kilograms, if they know that the total mass of a half dozen eggs is 0.226 kg.
- Have students place the decimal in each product. Ask them to explain how estimation helped them correctly place the decimal point in the product.
  a. 14 × 2.459 = 34426 b. 24.35 × 8 = 1948
- Ask students to identify which of the following is the best estimate for 13.7 × 9 and explain why.
  13.0 × 9 14.0 × 9 15.0 × 9 14.0 × 10 10.0 × 9

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N9: Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).** [CN, ME, PS, T]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
	N9 Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).	

ELABORATION

Guiding Questions:

• What do I want my students to learn?
• What do I want my students to understand and be able to do?

Students should realize that the convention for **order of operations** is necessary in order to maintain consistency of results in calculations.

The purpose of the order of operations is to ensure that the same answer is reached regardless of who performs the calculations. It is important to provide students with opportunities to explore solving problems in a variety of ways to recognize the different possible solutions. When more than one operation appears in an expression or equation, the operations must be performed in the following order:

- operations in brackets first;
- divide or multiply from left to right whichever operation comes first;
- add or subtract from left to right whichever operation comes first.

The acronym, “BEDMAS”**,** is a common memory device to recall the order of operations. It is important to stress that even though the “D” appears before the “M” and the “A” before the “S”, these pairs of operations are done in the order that they appear (multiplication or division, then addition or subtraction). The “E” represents exponents, however, this is not a concept Grade 6 students are expected to learn. It may be helpful to have students develop their own method of recalling the order of operations.

Students should be taught that **brackets** may also be referred to as **parentheses**. Some calculators have brackets that can be entered during calculations and the use of this function could be used by students. It is important that students recognize that most calculators will not use the order of operations when doing calculations. Students need to enter the digits on calculators following the correct order of operations.

When solving multi-step problems, it is important for students to recognize when it is appropriate to use technology. Students should be encouraged to use mental math and computational skills as much as possible. Students should be encouraged to solve problems mentally, such as $50 \times (12 \div 4)$.

SCO: **N9: Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).**
[CN, ME, PS, T]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Demonstrate and explain with examples why there is a need to have a standardized order of operations.
- Apply the order of operations to solve multi-step problems with or without technology, e.g., computer, calculator.

SCO: **N9: Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).**
[CN, ME, PS, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students work in groups to answer the following: 8 - 2 x 4 + 10 ÷ 2, then share their answers. Discuss why some found different answers and the need for rules so we all get the same answer. This could be extended by asking students where brackets could be placed to get the largest or smallest possible answer.
- Have students create word problems from given expressions (e.g., 4 × 10 + 8 × 3).
- Provide students with a variety of equations that do not have brackets and explore the possible solutions depending on where brackets are placed.
- Apply the rules for order of operations by modeling a variety of problem solutions. Students can check to determine whether their calculator follows the rules for order of operations. Depending on the type, calculators may yield different results. Students must be aware that most calculators will not use the order of operations to calculate equations automatically.

Suggested Activities

- Ask students to write a number sentence for the following: the total cost for a family with two parents and three children for theatre tickets if children's tickets cost $9 and adult tickets cost $12. When students write a number sentence such as, 3 × $9 + 2 × $12, ask if this solution makes sense:
  3 × $9 = $27 + 2 = $29 × $12 = $348.
- Have students write number sentences for the following problems and solve them using the order of operations. Consider solving the number sentences for a) and b) by ignoring the order of operations. Would the solution make sense in terms of the problem? Discuss.
  a. Ms. Janes bought the following for her project: 5 sheets of pressboard at $9 a sheet, 20 planks at $3 each, and 2 litres of paint at $10. What was the total cost?
  b. Three times the sum of $35 and $49 represents the total amount of Jim's sales on April 29. When his expenses, which total $75, were subtracted, what was his profit?
- Tell students that Billy had to answer the following skill-testing questions to win the contest prize. What are the winning answers? a. 234 × 3 – 512 ÷ (2 × 4) b. 18 + 8 × 7 – 118 ÷ 4
  Billy was told that the correct answer for "b" is 16, but Billy disagreed. What did the contest organizers do in solving the question which caused them to get 16 for the answer? Explain why you think they made that error.
- Ask students to explain why it is necessary to know the order of operations to compute 4 × 7- 3 × 6. Ask them to compare the solution of the previous problem with the solution of 4 × (7 - 3) × 6. Ask whether the solutions are the same or different and why.
- Provide students with a set of numbers and a target solution. Have students explore and discover where they can place operational symbols and brackets to achieve the solution. For example:
  3, 6, 3, 4. Solution = 11 3,6,3,4 Solution = 108 3,6,3,4 Solution = 6

Possible answers: 3 + (6 ÷3) × 4	(3 + 6) × (3 × 4)	(3 × 6)

Possible Models: calculators, two colour counters, computers

SCO: **N9: Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).**
[CN, ME, PS, T]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have students solve an order of operations expression and then describe what could have gone wrong if the order of operations steps were not followed (e.g., what might be an incorrect solution).
- Tell students that as a result of some faulty keys, the operation signs in these problems did not print. Use the information which is supplied to help determine which operations were used.
  a. (7 □ 2) □ 12 = 2 b. (12 □ 4) □ 4 = 7
- Tell students that because the shift key on the keyboard did not work, none of the brackets appeared in the following equations. If the student has the right answers to both problems, identify where the brackets must have been.
  a. 4 + 6 × 8 - 3 = 77 b. 26 - 4 × 4 - 2 = 18
- Have students use their calculator to answer the following question: Chris found the attendance reports for hockey games at the stadium to be 3419 and 4108. If tickets were sold for $12 each, and expenses for the stadium were $258 712, what was the profit for the two gamse? Have the students write out the equation to demonstrate their understanding of order of operations.
- Have students place brackets in the following equation to determine the various possible solutions.
  4 + 5 × 6 - 2 =

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **PR1: Demonstrate an understanding of the relationships within tables of values to solve problems.**
[C, CN, PS, R]
SCO: **PR2: Represent and describe patterns and relationships using graphs and tables**.
[C, CN, ME, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
PR1 Determine the pattern rule to make predictions about subsequent terms (elements).	**PR1** Demonstrate an understanding of the relationships within tables of values to solve problems. **PR2** Represent and describe patterns and relationships using graphs and tables.	**PR1** Demonstrate an understanding of oral and written patterns and their equivalent linear relations. **PR2** Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Mathematics is often referred to as the study of patterns, as they permeate every mathematical concept and are found in everyday contexts. The various representations of patterns including physical models, **table of values**, **algebraic expressions**, and graphs, provide valuable tools in making generalizations of mathematical relationships.

Patterns include **repeating** patterns and **growing** patterns. An example of a repeating pattern is 1, 2, 2, 1, 2, 2, 1, 2...). Growing patterns include **arithmetic** (adding or subtracting the same number each time) and **geometric** (multiplying or dividing the same number each time) situations. Patterns can be represented using concrete materials and pictures. Students should be able to describe these using words (three times a number, add five) and symbols ($3k + 5$), where numbers represent the quantity in each step of the pattern.

A table of values shows the relationship between pairs of numbers. Students should use tables to organize and graph the information that a pattern provides. When using tables, it is important for students to realize that they are looking for the relationship between two variables (**term number and term**). The **relationship** or **pattern rule** is what you do to the term number to get the term value. For example, the number pattern 1, 3, 5, 7, 9,… has the relationship where each number increases by two. The rule for this pattern is $2n$ -1. The table of values and pattern rule should be used to predict missing terms in the table or values not included in the table.

Term number (n)	1	2	3	4	5
Term (2n-1)	1	3	5	7	9

Input	Output
1	1
2	3
3	5
4	7
5	9

The analysis of graphs should include creating "stories" or real-world situations that describe the relationship depicted. Similarly, when constructing graphs, a story that matches the changes in related quantities should be included. When students are describing a relationship in a graph they should use language like: as this increases that decreases; as one quantity drops, the other also drops, etc.

Students should be able to derive a pattern rule and create a table of values for a given linear relationship and create a graph from a table of values. This concept is connected to outcomes SP1 and SP3.

SCO: **PR1: Demonstrate an understanding of the relationships within tables of values to solve problems.**
[C, CN, PS, R]
SCO: **PR2: Represent and describe patterns and relationships using graphs and tables**.
[C, CN, ME, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

PR1

- Generate values in one column of a table of values, given values in the other column and a pattern rule.
- State, using mathematical language, the relationship in a given table of values.
- Create a concrete or pictorial representation of the relationship shown in a table of values.
- Predict the value of an unknown term using the relationship in a table of values and verify the prediction.
- Formulate a rule to describe the relationship between two columns of numbers in a table of values.
- Identify missing elements in a given table of values.
- Identify errors in a given table of values.
- Describe the pattern within each column of a given table of values.
- Create a table of values to record and reveal a pattern to solve a given problem.

PR2

- Translate a pattern to a table of values and graph the table of values (limit to linear graphs with discrete elements).
- Create a table of values from a given pattern or a given graph.
- Describe, using everyday language, orally or in writing, the relationship shown on a graph.

SCO: **PR1: Demonstrate an understanding of the relationships within tables of values to solve problems.**
[C, CN, PS, R]
SCO: **PR2: Represent and describe patterns and relationships using graphs and tables**.
[C, CN, ME, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students identify the relationship, rule, and state the value for the 3rd and 12th terms for a given table.
- Develop tables with incorrect values and a correct pattern rule. Have students become “Data Detectives”, to find and correct the errors.
- Provide students with linear graphs with discrete data to analyze and have them create corresponding tables of values. Have them describe the relationship shown in the graph using words and symbols.

Suggested Activities

- Tell students that for their vacation a family drove for 5 hours the first day and travelled 450 km. The second day the family drove 8 hours and went 720 km. The last day they arrived in Las Vegas after driving 6 hours and 540 km. Have students create a table of values for this data, describe the pattern, and make a graph.
- Have students create the following pattern with counters, develop a table of values to display the information, write the relationship, and then graph it. Have students predict the value of unknown terms.

- Have students fill in the blanks in the tables below, describe the relationship for each and write the rule.

Numerator	?	2	3	?	5
Denominator	?	8	12	?	?

Side Length (cm)	1	2	3	4	5	6	
Perimeter (cm)	6	12	18		30		48

Input	Output
1	2
2	4
3	$\square$
4	8
$\square$	10

- Ask students to create a concrete and pictorial display of a table of values showing the balance in a bank account or the height of a plant as it grows. Have students graph the information.
- Describe a real-world situation that depicts a pattern. For example, a taxi ride costs $2.50 to start and then $0.40 for each kilometre. How much does it cost to travel 1 km? 2 km? 3 km?. Have students record the pattern, create a table of values, and graph the relationship. Have them determine the total cost of a 15 km trip.

Possible Models: pattern blocks, linking cubes

SCO: **PR1: Demonstrate an understanding of the relationships within tables of values to solve problems.**
[C, CN, PS, R]
SCO: **PR2: Represent and describe patterns and relationships using graphs and tables**.
[C, CN, ME, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

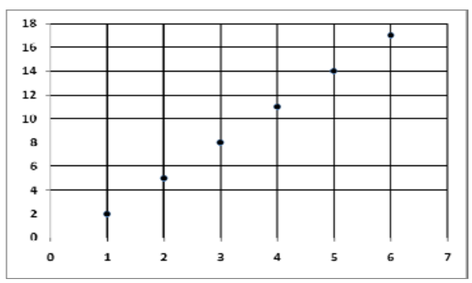
Whole Class/Group/Individual Assessment

- Ask the students to put the numbers 2, 4, 4, 5, 12, 20 and 40 in the correct spots in the tables of equivalent fractions shown below.

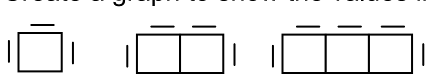
Numerator	1		3	
Denominator	4	8		16

Numerator	2		8	16
Denominator		10		

- Have the students create the table of values from a graph, such as the one below, and describe the pattern rule in words and symbols.

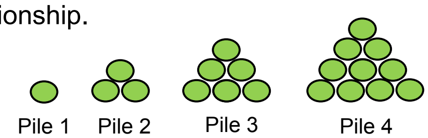


- Have students refer to the following table to answer these questions:
  a. What is the pattern rule for the number of chairs you would need for the tables? Explain your thinking.
  b. Use this rule to predict the number of chairs for 10 tables.
  c. Create a graph to show the values in the table.



Number of tables	Number of Chairs
1	4
2	6
3	8
4	10
5	12

- Provide a visual pattern such as the one below. Have students create and graph its table of values and describe the relationship. How many shapes would be needed to make the 8th pile?



FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **PR3: Represent generalizations arising from number relationships using equations with letter variables.** [C, CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
PR2 Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.	**PR3** Represent generalizations arising from number relationships using equations with letter variables.	**PR4** Explain the difference between an expression and an equation. **PR5** Evaluate an expression given the value of the variable(s).

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Mathematical patterns and number relationships occur in all areas of mathematics and can be generalized using **algebraic equations**. Previously, students have learned to build and model repeating and growing patterns, and then develop tables and charts to record them. Tables and charts are graphic organizers that allow students to see mathematical relationships. The next step is to be able to describe these patterns and relationships using an **expression**. An expression may include letters to represent the variable elements, numbers and operators (+, -, ×, ÷). It is important that students are able to create and generalize pattern rules to represent mathematical situations.

One opportunity for students to make generalizations arising from number relationships is when they explore perimeters and areas in the outcome SS3**.** One of the goals of this outcome is to make a connection between the concepts to create generalized formulas using variables.

Perimeter = $c + c + c$
= $3c$

Perimeter = $k + t + n$

Perimeter = $d + d + d + d$
= $4d$

Perimeter = $m + s + m + s$
= $2m + 2s$
Area = $m \times s$
= ms

Another example of a number relationship generalization is the **commutative property**. Earlier experiences with number combinations have led students to see that addition and multiplication are commutative: changing the order of the **addends** or **factors** does not change the answer. Using **variables** to represent the idea that order does not matter is a good way to describe the property (e.g., $a + b = b + a$ or $a \times b = b \times a$).

Word expressions and word problems should be used in this outcome to reinforce mathematical expressions (for example, “4 apples” could be expressed as “4*a*”, or “3 bananas and 2 pears is 5 fruit” could be “3*b* + 2*p* = 5”, etc.).

Students should also have opportunities to develop mathematical relationships and expressions from the patterns found in tables such as those investigated in PR1 and PR2.

SCO: **PR3: Represent generalizations arising from number relationships using equations with letter variables.**
[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Write and explain the formula for finding the perimeter of any regular polygon.
- Write and explain the formula for finding the area of any given rectangle.
- Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication, e.g., $a + b = b + a$ or $a \times b = b \times a$.
- Describe the relationship in a given table using a mathematical expression.
- Represent a pattern rule using a simple mathematical expression, such as $4d$ or $2n + 1$.

SCO: **PR3: Represent generalizations arising from number relationships using equations with letter variables.**
[C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

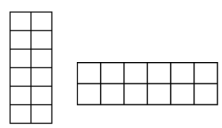
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students examine the perimeter of regular polygons with various side lengths. They could record the data for a regular hexagon as shown in the table below.

Side length (n) (cm)	1	2	3	4	5	6
Perimeter (P) (cm)	6	12	18	24	30	36

  The next step is to have students generalize the pattern they have found for the perimeter of regular hexagons, stating the pattern rule as an algebraic equation: P(hexagon) = 6n
  Other types of generalizations can be developed through measurement and pattern tables as students explore the perimeters of other regular polygons and areas of rectangles in SS3.
- Reinforce the concept that multiplication is commutative by having students build an array model of a multiplication fact using linking cubes or tiles. Have them turn the model to show the factors in a different order, illustrating the concept that the same product results. The final step is to have students replace the factors with variables.

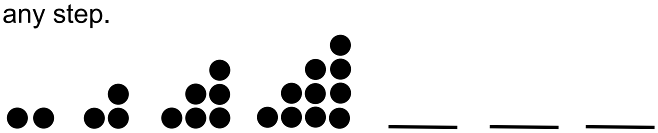


$a \times b = b \times a$

$6 \times 2 = 2 \times 6$

Suggested Activities

- Play "Guess My Rule". Describe a number pattern and have students create the mathematical expression that matches. For example: "I double every time" or "Divide me in half and add three every time."
- Provide a table of values and have students generalize the pattern rule and record it as an algebraic equation.
- Provide students with pictures or models of the first three steps of some growing patterns. Have students extend the pattern for several more steps, record the pattern in a table, and look for the relationship. Have them write the relationship as a formula and use the formula to predict entries at any step.



Term Number	Term
1	2
2	3
3	6
4	10

Possible Models: pattern blocks, colour tiles, linking cubes, 2-D shapes

SCO: **PR3: Represent generalizations arising from number relationships using equations with letter variables.**
[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

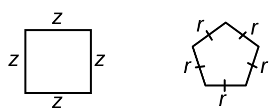
Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

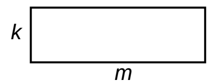
- Have students create an expression for the following patterns:
  - a number doubles;
  - a number triples and 2 is added every time.
- Provide students with a number of equations such as 27 + 15 = *n* + 27. Observe whether the students misinterpret the meaning of the variable, of the meaning the equal sign, or the commutative property by answering 42. Include multiplication equations as well (*see* also PR4).
- Have students explain how the following two expressions are the same and different. Explain using models, pictures, and words.
  m × n n × m
- Provide the following table and ask the student to generalize the relationship with an expression.

Side length (cm)	1	2	3	4	5
Perimeter (cm)	4	8	12	16	20

- Have students write and explain the formula for finding the perimeter of any square, pentagon, or other regular polygons using variables.



- Have students write and explain the formula for finding the area of any given rectangle using variables.



FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **PR4: Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
PR2 Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.	**PR4** Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.	**PR4** Explain the difference between an expression and an equation. **PR5** Evaluate an expression given the value of the variable(s).

ELABORATION

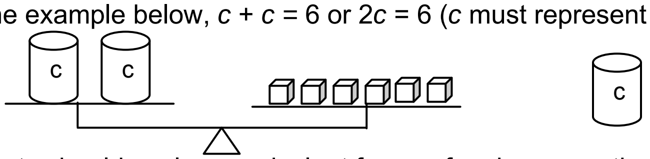
Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students have been exploring the concept of equality since Grade 2 and solving equations in a basic form since Grade 3. A misconception for some students may be that the equal sign indicates an answer. They will need further practice and reinforcement in Grade 6 to view the equal sign as a symbol of **equivalence** and balance, and represents a **relationship**, not an operation.

Through the use of balance scales and concrete representations of equations, students will see the equal sign as the midpoint or balance, with the quantity on the left of the equal sign is the same as the quantity on the right. When the quantities balance, there is **equality**. When there is an imbalance, there is **inequality**. The work in Grade 6 extends this concept so that students discover that any change to one side must be matched with an equivalent change to the other side in order to maintain the balance. For example, if four is added to the left side of the equation, four must be added to the right side in order to preserve the equality.

In Grades 3 and 4, **variables** are represented using a variety symbols such as circles and triangles. In Grade 5, students were introduced to using letters as variables. However, students may have the misconception that $7w + 22 = 109$ and $7n + 22 = 109$ will have different solutions because the letter representing the variable has changed. Also they may see letters as objects rather than numerical values. Conventions of notations using variables may also produce misunderstandings. For example, $j \times z$ is written as jz, but 3×5 cannot be written as "35" and $2g$, where $g = 4$ means 2 times *4*, not 24.

When using variables, or representing variables using concrete objects, such as paper bags or boxes, students need to be directly taught that if the same variable, or object, is used repeatedly in the same equation, then there is only one possible solution for that variable or unknown.
For the example below, $c + c = 6$ or $2c = 6$ (c must represent the same number).



c must be worth 3

Students should explore equivalent forms of a given equation by applying the **preservation of equality** and verify using concrete materials on a balance. They should draw and record the original equation and then add the same amount to both sides. Students should observe that no matter how much they add, the scale will remain balanced as long as they add the same amount on each side. This will help students observe how the equality of the two sides of the equation is preserved. This type of investigation should be repeated to explore subtracting the same amount from both sides, multiplying both sides by the same factor (e.g., double each amount), or dividing both sides by the same divisor.

SCO: **PR4: Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Model the preservation of equality for addition using concrete materials, such as a balance or using pictorial representations and orally explain the process.
- Model the preservation of equality for subtraction using concrete materials, such as a balance or using pictorial representations and orally explain the process.
- Model the preservation of equality for multiplication using concrete materials, such as a balance or using pictorial representations and orally explain the process.
- Model the preservation of equality for division using concrete materials, such as a balance or using pictorial representations and orally explain the process.
- Write equivalent forms of a given equation by applying the preservation of equality and verify using concrete materials, e.g., $3b = 12$ is the same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$.

SCO: **PR4: Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

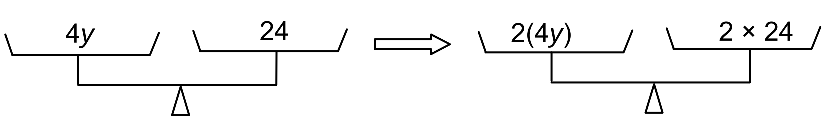
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

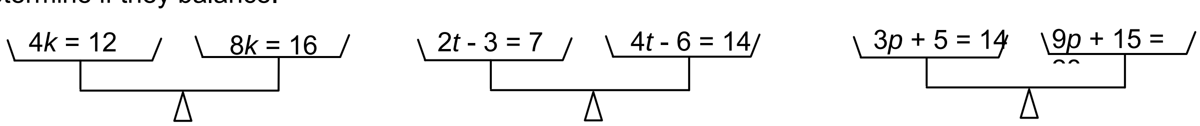
- Use a balance to model equations with bags to represent variables (unknown amounts) and linking cubes or blocks to represent numbers.
- Build known quantities on balance scales to model equality and also model how changes to one side must be matched with equivalent changes on the other. For example, model 3 cubes plus 5 cubes on one side and 8 cubes on the other. Have students record the equation. Then model adding 4 to both sides, subtracting 2 from both sides, doubling both sides, halving both sides, etc. Have the students record the equations.
- Model concrete examples of equations that include a variable, such as $3 + x = 10$. Model and record the preservation of equality when 5 is added to each side (e.g., $3 + x + 5 = 10 + 5$). Also explore the preservation of equality using subtraction, multiplication and division on both sides of the equation.
- Explore the preservation of equality for multiplication by determining whether each side of the equation was multiplied by the same amount. For example, $2r + 3 = 11$ and $6r + 9 = 33$ would be equivalent because all terms in the first equation were multiplied by 3 (tripled). Use a balance to verify.
- Use websites such as the *National Library of Virtual Manipulatives* or *Learn Alberta* to provide opportunities to further explore this outcome: www.learnalberta.ca/content/mesg/html/math6web/lessonLauncher.html?lesson=m6lessonshell11.swf

Suggested Activities

- Extend the activity of “Tilt or Balance” game (Van de Walle & Lovin, vol. 3, 2006, p. 279) to include adding and subtracting variables.
- Provide illustrations of pan balances that show equal expressions. Ask students to draw and record the shown equation, then draw and record the results when adding the same amount to both sides, subtracting the same amount from both sides, multiplying both sides by the same factor, and dividing both sides by the same divisor.



- Provide a variety of illustrations of pan balances with expressions on each side. Ask students to determine if they balance.



Possible Models: balance scales, linking cubes, base ten blocks, number lines, objects to represent the variables such as pattern blocks, colour tiles, or geometric solids

SCO: **PR4: Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

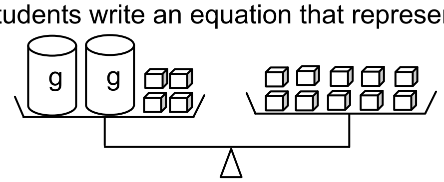
Look back at what you determined as acceptable evidence.

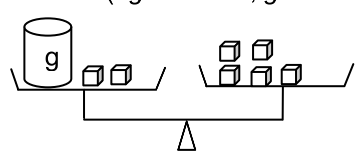
Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask students to model the following equations using balance scales and various materials.
  Examples: $12 + 2s = 18$
  $17 = 5b - 3$
  $3p = 18 \div 2$
- Have students determine whether the following pairs of equations are equivalent.
  $14 - s = 8$ and $9 - s = 3$
  $30k = 90$ and $5k = 15$
  $12 = 17 - j$ and $10 = 19 - j$
  $18 = 16 + s$ and $9 = 8 + s$
- Have students model and write two equations that are equivalent to $4b = 12$. Explain how they know they are equivalent.
- Ask students if $2g + 3 = 7$ and $3g + 4 = 8$ are equivalent equations. Explain why or why not? Use a model to represent each equation.
- Have students write an equation that represents each model ($2g + 4 = 10$; $g + 2 = 5$):

  a. 

  b. 

  - Are the equations for these two scales equivalent? How do you know?
  - Model and record what will happen if you add 2 cubes to each side of the balance in “a”. Draw the results. Repeat for subtracting 2 from each side of “a”.
  - Model, draw and record what happens if you multiply both sides of “b” by 3.
  - Model, draw and record what happens if you multiply both sides of “a” by 3.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS1: Demonstrate an understanding of angles by:**
- **identifying examples of angles in the environment**
- **classifying angles according to their measure**
- **estimating the measure of angles using 45°, 90° and 180° as reference angles**
- **determining angle measures in degrees**
- **drawing and labelling angles when the measure is specified.**

[C, CN, ME, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

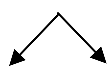
Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
SS3 Demonstrate an understanding of area of regular and irregular 2-D shapes by: recognizing that area is measured in square units**;** selecting and justifying referents for the units cm^2 or m^2**;** estimating area by using referents for cm^2 or m^2**;** determining and recording area constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.	**SS1** Demonstrate an understanding of angles by: • identifying examples of angles in the environment • classifying angles according to their measure • estimating the measure of angles using 45°, 90 and 180° as reference angles • determining angle measures in degrees • drawing and labelling angles when the measure is specified.	**SS1** Demonstrate an understanding of circles by: describing the relationships among radius, diameter and circumference of circles; relating circumference to pi; determining the sum of the central angles; constructing circles with a given radius or diameter; solving problems involving the radii, diameters and circumferences of circles.

ELABORATION

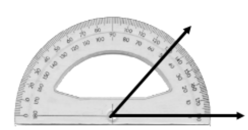
Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students have been previously introduced to the idea of angles during their study of polygons but in Grade 6 the properties of angles are explored in greater depth. Frequently, angles are defined as the meeting of two **rays** (**arms**) at a common **vertex**. It is more useful, however, for students to conceptualize an angle as a **turn** and the measure of the angle as the amount of turn. It is important for students to understand that:
- a larger angle corresponds to a greater turn from the starting position;
- the length of the rays (arms) of the angle does not affect the turn amount and, therefore, does not affect angle size;
- the orientation of an angle does not affect its measurement or classification.



It is also important that students learn the different types of angles and be able to **classify** them as **acute** (less than 90°), **right** (exactly 90°), **obtuse** (91° to 180°), **straight** (exactly 180°), **reflex** (more than 180°).

Students should learn how to use a **protractor** to measure angles accurately. When drawing or measuring angles, students need to be reminded that the centre point of the protractor needs to be lined up with the vertex of the angle, and the 0^o line of the protractor must line up exactly with one ray of the angle. Students typically use protractors with double scales and will need to learn how to determine which set of numbers to use in a given situation. This is best accomplished by first having the student estimate the size of the angle with known benchmark angles such as 45°, 90° and 180° and then decide which reading makes the most sense. For example, the angle shown below is obviously an acute angle, and therefore its measure is 50°, not 130°.



SCO: **SS1: Demonstrate an understanding of angles by:**

- **identifying examples of angles in the environment**
- **classifying angles according to their measure**
- **estimating the measure of angles using 45°, 90° and 180° as reference angles**
- **determining angle measures in degrees**
- **drawing and labelling angles when the measure is specified.**

[C, CN, ME, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Provide examples of angles found in the environment.
- Classify a given set of angles according to their measure, e.g., acute, right, obtuse, straight, reflex.
- Sketch 45°, 90° and 180° angles without the use of a protractor, and describe the relationship among them.
- Estimate the measure of an angle using 45°, 90° and 180° as reference angles.
- Measure, using a protractor, given angles in various orientations.
- Draw and label a specified angle in various orientations using a protractor.
- Describe the measure of an angle as the measure of rotation of one of its sides.
- Describe the measure of angles as the measure of an interior angle of a polygon.

SCO: **SS1: Demonstrate an understanding of angles by:**
- **identifying examples of angles in the environment**
- **classifying angles according to their measure**
- **estimating the measure of angles using 45°, 90° and 180° as reference angles**
- **determining angle measures in degrees**
- **drawing and labelling angles when the measure is specified.**

[C, CN, ME, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

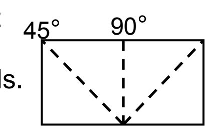
Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

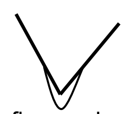
Consider the following strategies when planning lessons:

- Have students identify angles in a variety of real life contexts (e.g., angles formed by the two hands of a clock, by the intersection of two roads, and by the blades of scissors or hedge clippers).
- Explore the similarities and differences between rulers and protractors. Students should recognize that protractors work similarly to rulers as they measure the amount of rotation between the arms. They should also note that protractors have two numbers at each measurement point.
- Show students angles (with arms of different lengths) in various positions and of different sizes. Ask them to estimate each (e.g., almost 45°, 90°, 180°, etc.).
- Have students stand with their arms closed on top of each other pointing out in the same direction to one side. This shows (0°)**.** Then have them raise one arm up until it points directly up (90°), then continue rotating their arm until their arms are out straight to make a straight angle (180°).
- Use children's literature such as *Sir Cumference and the Great Knight of Angleland* by Cindy Neuschwander to explore protractors and the different types of angles.
- Have students create their own non-standard protractors. Provide the students with pieces of translucent paper (tracing paper or waxed paper). Have them fold the paper in half, forming a right angle or square corner. Explain that angles are measured in degrees and that a right angle is 90 degrees. Ask them to fold once again and determine and name the new angles created by the folds. Discuss the measurement of these folds and how they can assist with estimation of angle sizes.



Suggested Activities

- Have students investigate angles in various shapes, using the corner of a piece of paper as a reference for right angle. Does it fit the angle of the shape or is the angle greater/less than the corner of the paper?
- Have students make various angles with pipe cleaners or geo-strips (e.g., almost a right angle, about 45°, a right angle, a straight angle, a reflex angle).
- Ask students to explore the angles in the six different pattern blocks. Which blocks have only acute angles? Only obtuse angles? Both acute and obtuse angles? Only right angles?
- Display different times one at a time on a clock. Ask the student to name and describe the angle made by the hands.
- Ask the student to measure the angles found in various letters of the alphabet.
- Ask the student where acute, right, obtuse, straight, and reflex angles could be identified in the classroom.



reflex angle

Possible Models: geo-strips, Power Polygons, fraction circles, geoboards, pattern blocks, clock hands, 2-D shapes and 3-D objects

SCO: **SS1: Demonstrate an understanding of angles by:**
- **identifying examples of angles in the environment**
- **classifying angles according to their measure**
- **estimating the measure of angles using 45°, 90° and 180° as reference angles**
- **determining angle measures in degrees**
- **drawing and labelling angles when the measure is specified.**

[C, CN, ME, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

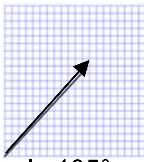
Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

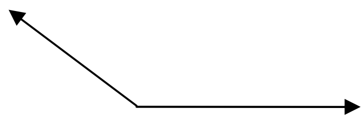
Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask students to combine two or more pattern blocks to make examples of acute, right, straight, and obtuse angles. Have them record by tracing each one on paper.
- Tell students that the hands of a clock are forming a given angle (such as 45°). Ask what could be the time.
- Show students the diagram below and ask why it is easy to tell that it is 45°.



- Show students an angle of, for example 135°, and tell him/her that someone said that it was 45°. Ask them to explain how they think such an error could be made.
- Provide students with various angles and have them measure each with a protractor.
- Have students draw angles with specified measures using a protractor.
- Ask students how a 90° angle could be used to construct a 45° angle?
- Have students identify angles in objects in the classroom and name the types of angles on the shapes. Have students estimate the sizes of the angles.
- Have students identify angles in various 2-D polygons and on the faces of 3-D shapes and name the types of angles on the shapes. Have students estimate the sizes of the angles.
- Tell students that Trevor measured the angle below and said it measured 50°. What was his error?



FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SS2: Demonstrate that the sum of interior angles is:** • **180° in a triangle** • **360° in a quadrilateral.** [C, R]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
SS6 Identify and sort quadrilaterals, including: rectangles; squares; trapezoids; parallelograms; rhombuses according to their attributes.	**SS2** Demonstrate that the sum of interior angles is: • 180° in a triangle • 360° in a quadrilateral.	**SS1** Demonstrate an understanding of circles by: describing the relationships among radius, diameter and circumference of circles; relating circumference to pi; determining the sum of the central angles; constructing circles with a given radius or diameter; solving problems involving the radii, diameters and circumferences of circles.

ELABORATION

Guiding Questions:

• What do I want my students to learn?

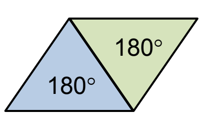
• What do I want my students to understand and be able to do?

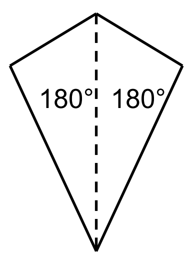
In previous grades, students have investigated some of the attributes of polygons, such as side lengths and vertices. Students will build on these experiences in Grade 6 as angles and other properties are investigated in greater depth. It is recommended that SS1 and SS4 be taught prior to this outcome, so that students are familiar with the measurement of angles, the different types of triangles, and the vocabulary to name and describe them.

Through explorations, students should discover that the angles of a triangle add to 180°. This can be done using paper models and/or dynamic geometry software such as *Geometer's Sketchpad* (http://dynamicgeometry.com/) or Smart Board *Notebook*. Different types of triangles (acute-angled, isosceles, obtuse-angled, equilateral, etc.) need to be used so that students discover that this property applies to all types of triangles.

Once students have an understanding of this property, it would be beneficial to have students to measure the interior angles of the triangles using a protractor and find the sum. Students may notice that in some cases their sum does not equal 180° exactly, but is very close. It is important that students recognize the potential for human error in measurement.

Exploration of the angle properties of triangles should be extended to **quadrilaterals** by concretely investigating the relationship between triangles and quadrilaterals. Students should discover that two triangles can be combined to create a quadrilateral and conclude that the sum of the angles is 360° (180° + 180°).





SCO: **SS2: Demonstrate that the sum of interior angles is:**

- **180° in a triangle**
- **360° in a quadrilateral.**

[C, R]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Explain, using models, that the sum of the interior angles of a triangle is the same for all triangles.
- Explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals.

SCO: **SS2: Demonstrate that the sum of interior angles is:**
- **180° in a triangle**
- **360° in a quadrilateral.**

[C, R]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

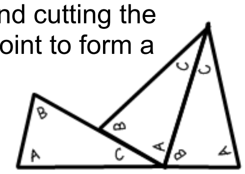
Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

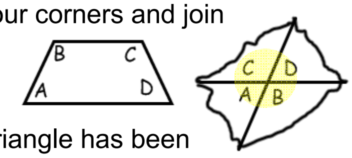
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

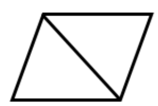
- Have the student draw a triangle of any type and label its angles 1, 2, 3. Cut it out. Then have the student tear off the three angles and place the three vertices together to form a 180° angle. Have students measure and record the three angles and find the sum.
- Have the student cut out three congruent triangles by stacking three sheets of paper and cutting the three shapes at once. Rotate the triangles so the three different vertices meet at one point to form a 180° angle. Use graphic software or interactive whiteboards and repeat this activity.



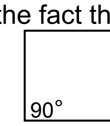
- Cut out a quadrilateral and label the four vertices. Have the student tear off the four corners and join the vertices together. Highlight the 360° sum.



- Have students draw and cut out a quadrilateral once the sum of the angles of a triangle has been explored and determined. Have them determine that a quadrilateral can be made up of two triangles, and the sum of the angles of those two triangles equals 360°.



- Explore how the characteristics of a square are helpful for students to remember the fact that every quadrilateral has a sum of angles equal to 360°.



Suggested Activities

- Have students each draw a variety of different triangles. Have them measure, record, and add the angles of each one. Have them discuss their findings until they reach the conclusion that the sum of the angles of *any* triangle is 180°. Repeat the above activity using a variety of quadrilaterals.
- Provide a variety of triangles with the measures of two angles shown. Students must find the measure of the third angle using their understanding of the sum of the angles of a triangle (without a protractor).
- Ask students to predict the interior angle of an equilateral triangle, and then check by measuring with a protractor.
- Provide students with a variety of quadrilaterals with the measures of three of the angles given. Students must find the measure of the fourth angle without a protractor.
- Ask students to predict the interior angle of an equilateral triangle, and then check by measuring with a protractor.

Possible Models: protractors, pattern blocks, tangrams, attribute blocks, Power Polygons™

SCO: **SS2: Demonstrate that the sum of interior angles is:**
- **180° in a triangle**
- **360° in a quadrilateral.**

[C, R]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

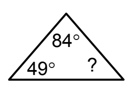
Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

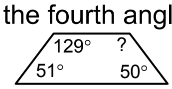
Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask student whether a triangle can have more than one obtuse angle? Why or why not? Explain using numbers, pictures, and/or words.
- Ask students whether a triangle can have two right angles? Why or why not? Explain using numbers, pictures, and/or words.
- Ask students to explain how knowing that the sum of the angles in a triangle equals 180° helps them to know the sum of the angles in a quadrilateral. Have students explain their thinking using numbers, pictures, and/or words.
- Have students solve to find the measure of the third angle of a triangle when the measures of the other two angles are given.



- Have students solve to find the measure of the fourth angle of a quadrilateral when the measures of the other three angles are given.



FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SS3: Develop and apply a formula for determining the:**
- **perimeter of polygons**
- **area of rectangles**
- **volume of right rectangular prisms.**

[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
SS3 Demonstrate an understanding of volume by: selecting and justifying referents for cm^3 or m^3 units; estimating volume by using referents for cm^3 or m^3; measuring and recording volume (cm^3 or m^3) constructing rectangular prisms for a given volume.	**SS3** Develop and apply a formula for determining the: • perimeter of polygons • area of rectangles • volume of right rectangular prisms.	**SS2** Develop and apply a formula for determining the area of: parallelograms; triangles; circles.

ELABORATION

Guiding Questions:

• What do I want my students to learn?

• What do I want my students to understand and be able to do?

The basic concepts of perimeter, area, and volume have been introduced and explored in previous grades. Students have estimated and worked with both non-standard and standard units. The emphasis for Grade 6 is to have students discover the *most efficient strategies* for finding these measures. These explorations should eventually elicit from students the traditional **formulas** for **perimeter of polygons**, **area of rectangles**, and **volume of right rectangular prisms**. This outcome is closely connected to PR3 where students use letter variables to express a formula.

As a result of prior experiences, students should conceptualize perimeter as the total distance around a closed object or figure. They might observe that, for certain polygons, the perimeter is particularly easy to compute.
- Equilateral triangle: the perimeter is 3 times the side length.
- Square: the perimeter is 4 times the side length.
- Rectangle: the perimeter is double the sum of the length and the width.

Students will be familiar with the concept of area from Grade 4, where they found the area of rectangles using standard units. “From earlier work with multiplication and the array meaning or model of multiplication, students will know that, to determine the total number of squares, you multiply the number of rows of squares by the number of squares in each row” (Small, 2008, p. 398). Students need to have many opportunities to experiment with the relationships among length, width, and area to develop their own formulas for area of rectangles (remind students that a square is a special type of rectangle).

Volume has been studied in Grade 5. Students should recognize volume as:
- the amount of space taken up by a 3-D object; or
- the amount of cubic units required to build and fill the object.

Students should also recognize that each of the three dimensions of the prism affects the volume of the object. Development of the concept of using the **area of the base** as part of the formula for volume of a right rectangular prism will be helpful for work in later grades as volume of other 3-D objects is explored.

SCO: **SS3: Develop and apply a formula for determining the:**
- **perimeter of polygons**
- **area of rectangles**
- **volume of right rectangular prisms.**

[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Explain, using models, how the perimeter of any polygon can be determined.
- Generalize a rule (formula) for determining the perimeter of polygons, including rectangles and squares.
- Explain, using models, how the area of any rectangle can be determined.
- Generalize a rule (formula) for determining the area of rectangles.
- Explain, using models, how the volume of any right rectangular prism can be determined.
- Generalize a rule (formula) for determining the volume of right rectangular prisms.
- Solve a given problem involving the perimeter of polygons, the area of rectangles and/or the volume of right rectangular prisms.

SCO: **SS3: Develop and apply a formula for determining the:**
- **perimeter of polygons**
- **area of rectangles**
- **volume of right rectangular prisms.**

[C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

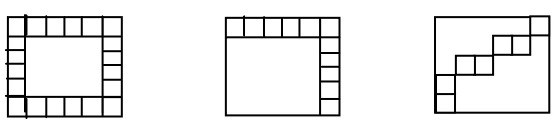
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Provide pictures of many regular polygons, with the measure of one side provided for each. Have students explore to find the most efficient method for finding the perimeters of each. Lead students to discover that “side + side + side + side…” is inefficient when multiplication can be used instead. Repeat the activity with rectangles and parallelograms.
- Provide students with graphics of many rectangles, including squares, in which the square units are shown and the length and width measures are given. Ask students to find the most efficient way to find the areas of each. Begin with small areas, such as 2 cm × 3 cm, and help students relate these rectangles to the array model for multiplication.
- Have students create many different rectangles, including squares, on grid paper. Have them find and record the length, width, and area for each (by counting the squares, if necessary). They should record their findings in chart form, so they can look for relationships in the table among the length, width, and area for each. Lead students to develop the formula: *length* × *width* (Small, 2008, p. 398).
- Have students build a variety of right rectangular prisms. On a chart, have them record the length and width of the base and the height, as well as the volume. Have students look for relationships among these measures and lead students toward developing the formula.

Suggested Activities

- Provide students with a variety of rectangles with incomplete grids. Have them apply the formula to determine their areas.



- Provide students with regular polygons to explore to find patterns between side lengths and create a rule (formula) for each to calculate the perimeter.
- Present students with rectangular prisms constructed out of linking cubes. Have them calculate the volume. Determine if the student uses multiplication rather than counting cubes.
- Provide the students with linking cubes and have them build cubic structures of different sizes. In each case, ask them to record the various side lengths and volumes in a table.
- Have students estimate the volume of a cube with a side length of 2.5 units. Extend to other side lengths.

Possible Models: rulers, grid paper, base ten blocks, linking cubes, 1 cm cubes

SCO: **SS3: Develop and apply a formula for determining the:**
- **perimeter of polygons**
- **area of rectangles**
- **volume of right rectangular prisms.**

[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

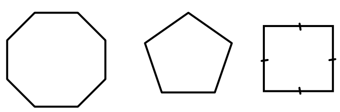
Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Tell students that the perimeter of a triangle is 15 cm. Have them describe and draw the possible side lengths. (Note: if outcome SS4 has been done, the type of triangle can be specified – scalene, isosceles, etc.)
- Ask: "How can a formula be used to determine the perimeter of the following regular polygons?"



- Provide students with area problems to solve such as the following:
  - A teen mowed two lawns. One lawn was 10 m by 12 m, and the other was 15 m by 10 m. The teen charges \$3.00 for each 10 m^2. How much was charged for the two lawns?
  - Zack needs to mail a present to his cousin. The box is 24 cm long, 15 cm wide, and 5 cm high. The shipper charges \$0.75 for each 1 cm^3 and \$3.00 for the total mass. How much will it cost to ship the package?
- Provide students with the dimensions of a real world container that is a rectangular prism (e.g., a carton, a box, a popcorn bag, etc.). Ask students to find the perimeter and area of each face. Students should also determine the volume for the prism. Ask students to determine the possible dimensions if the object needed to hold twice as much.
- Explain, using numbers, pictures, and/or words, why a rectangular prism that is 5 cm by 3 cm, with a height of 4 cm must have a volume of 60 cm^3.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS4: Construct and compare triangles, including:**

- **scalene**
- **isosceles**
- **equilateral**
- **right**
- **obtuse**
- **acute**

in different orientations.
[C, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence of Outcomes

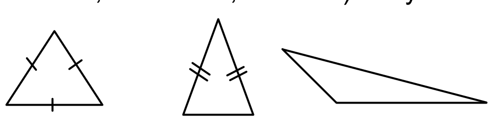
Grade Five	Grade Six	Grade Seven
SS5 Describe and provide examples of edges and faces of 3-D objects and sides of 2-D shapes that are: parallel; intersecting; perpendicular; vertical or horizontal.	**SS4** Construct and compare triangles, including: • scalene • isosceles • equilateral • right • obtuse • acute in different orientations.	**SS3** Perform geometric constructions, including: perpendicular line segments**;** parallel line segments**;** perpendicular bisectors**;** angle bisectors.

ELABORATION

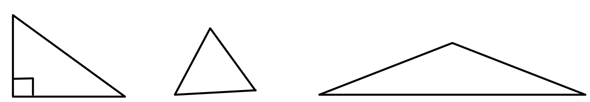
Guiding Questions:

• What do I want my students to learn?
• What do I want my students to understand and be able to do?

Students need to realize triangles can be sorted either by the length of their sides (**equilateral**, **isosceles**, **scalene**) or by the size of their angles (**right**, **acute**, **obtuse**).



equilateral isosceles scalene



right acute obtuse

Students should explore why there are only three possible classifications by side length. It should be discovered that triangles cannot be classified according to one "equal side", but there needs to be zero, two, or three equal sides. Similar discussion may be held around the reasons for the three different types of triangles in the set of classifications by angle size. For example, a triangle cannot have more than one obtuse angle (greater than 90°) as the angles in a triangle add up to 180°. Once these two sets of classification have been studied, teachers should extend students' knowledge to explore how a triangle may fall into two categories at the same time (e.g., a right scalene triangle, an obtuse isosceles triangle, etc.).

Students have not used the term **congruent** before this point, although they have had experience comparing and matching 2-D shapes based on attributes. It would be helpful to introduce the symbol for congruence (≅). Also ensure students are aware of the meaning of the equivalence marks on the sides of polygons as shown on the triangles above.

It is important to provide students with frequent opportunities to explore and create different types of triangles. Students should recognize that being given three sides or two angles and one side length or two sides and one angle will result in a unique triangle.

SCO: **SS4: Construct and compare triangles, including:**
- **scalene**
- **isosceles**
- **equilateral**
- **right**
- **obtuse**
- **acute**

in different orientations.
[C, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Sort a given set of triangles according to the length of the sides.
- Sort a given set of triangles according to the measures of the interior angles.
- Identify the characteristics of a given set of triangles according to their sides and/or their interior angles.
- Sort a given set of triangles and explain the sorting rule.
- Draw a specified triangle, e.g., scalene.
- Replicate a given triangle in a different orientation and show that the two are congruent.

SCO: **SS4: Construct and compare triangles, including:**
- **scalene**
- **isosceles**
- **equilateral**
- **right**
- **obtuse**
- **acute**

in different orientations.
[C, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Provide a variety of triangles and have students compare and measure the angles so that they are able to discover these patterns: a) all angles in equilateral triangles are equal; b) two angles in isosceles triangles are equal; and c) all angles in scalene triangles are different.
- Have students test for congruency by placing one shape on top of another to see if the outlines match exactly.
- Give students cards with examples of right, acute, and obtuse triangles on them. Ask them to sort them into three groups by the nature of their angles and share how they were sorted. Attach the names for these classifications to the students' groups.
- Use Venn diagrams or Carroll diagrams to help organize sorted triangles.

Suggested Activities

- Prepare pictures on cards or cutouts of several examples of different kinds of triangles. Ask the students to sort them into three groups and provide their sorting area. Often, they will sort triangles by how their sides look, without knowing the actual names. If so, this will lead to a focus on measuring and comparing the sides, and noting common properties to which the names equilateral, isosceles, and scalene can be attached. (If not, the teacher may sort them, ask the students to determine the sorting rule, and do other explorations.)
- Identify everyday examples of each type of triangle; yield sign, bridges, the ends of a Toblerone bar, other support items, ladder against a wall. Students should also examine familiar materials in the classroom, such as pattern blocks and tangrams.
- Provide pairs of students with two 6 cm straws, two 8 cm straws, and two 10 cm straws. Have them investigate the triangles they can make using 3 straws at a time and complete a table with their results. This activity could be varied by using toothpicks or geo-strips.

Straw Sizes Used	Type of Triangle

- Read *The Greedy Triangle* by Marilyn Burns and discuss the types of triangles shown in the book.
- Provide students with paper cut-outs of various types of triangles. Have them explore how many different orientations of the same triangle they can find and trace.
- Have students draw a triangle on tracing paper and classify it. Have them fold the paper in order to trace the shape several different ways to create congruent triangles in other orientations.

Possible Models: geoboards, grid paper, geo-strips, tangrams, straws or pipe cleaners

SCO: **SS4: Construct and compare triangles, including:**

- **scalene**
- **isosceles**
- **equilateral**
- **right**
- **obtuse**
- **acute**

in different orientations.
[C, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Provide students with a set of triangles (be sure to include a variety of different types). Have them sort the triangles first according to the length of the sides (equilateral, isosceles, scalene) and explain their sorting rule. Repeat the task having students sort the triangles according to the measures of the angles (right, acute, obtuse) and explain their sorting rule.
- Have students draw the following or other examples of triangles that can be classified more than one way (e.g., a scalene right triangle; an isosceles, acute-angle triangle).
- Have students construct specific triangles on their geoboards and record them on dot paper (e.g., an acute triangle that has one side using five pins; a right triangle that is also isosceles; an obtuse triangle that has one side using five pins).
- Provide students with a geoboard and dot paper. Have them create and draw 2 different:
  - scalene triangles
  - isosceles triangles
  - right triangles
  - equilateral triangles
  - acute triangles
  - obtuse triangles
- Have students draw various triangle types with specific properties, such as:
  - an obtuse triangle with an angle of 130°;
  - a triangle with 3 cm and 4 cm sides that form a right angle;
  - an equilateral triangle with 10 cm sides;
  - an obtuse triangle with a 110° angle and one 5 cm side.
- Tell students that one side of a triangle is 20 cm. What might the lengths of the other two sides be for each of the followings kinds of triangles?
  - isosceles
  - scalene
  - equilateral

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SS5: Describe and compare the sides and angles of regular and irregular polygons.**
[C, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
SS5 Describe and provide examples of edges and faces of 3-D objects and sides of 2-D shapes that are: parallel; intersecting; perpendicular; vertical or horizontal. **SS6** Identify and sort quadrilaterals, including: rectangles; squares; trapezoids; parallelograms; rhombuses according to their attributes.	**SS5** Describe and compare the sides and angles of regular and irregular polygons.	**SS3** Perform geometric constructions, including: perpendicular line segments**;** parallel line segments**;** perpendicular bisectors**;** angle bisectors.

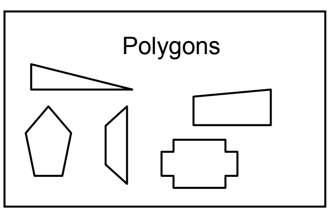
ELABORATION

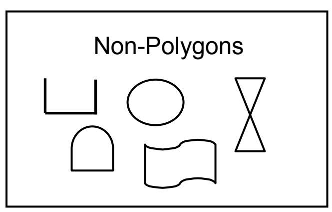
Guiding Questions:

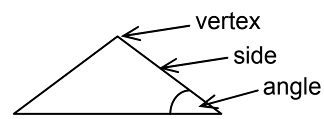
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students learned in Grade 3 the names of common polygons and were introduced to the concept of regular and irregular polygons. The focus of Grade 6 is to include all of the **properties** of **sides** and **angles** of shapes in the classification process. Teachers need to provide students with a variety of sorting activities of 2-D shapes and questions to help guide their investigations.

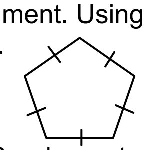
Polygons are closed 2-D shapes with three or more straight sides. The sides meet only at the **vertices**. A key property of polygons is that the number of sides is always equal to the number of vertices. Shapes that are missing one or more of these attributes are considered **non-polygons**. It is important that students focus on these attributes using proper vocabulary to determine whether the shape is a polygon. A common misconception is to think that triangles and quadrilaterals are not polygons since they have other names.



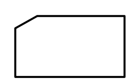




In Grade 6, students will extend their knowledge to include both regular and irregular polygons. **Regular polygons** have all sides and angles equal (e.g., pattern blocks: equilateral triangles, squares, hexagons). **Irregular polygons** do not have all sides or angles that are the same size. Students should be given opportunities to explore both regular and irregular polygons in their environment. Using the attributes of polygons, students should be able to sort into regular or irregular polygons.



Regular pentagon



Irregular pentagon

It is also important for students to investigate the concept of congruence by superimposing the shapes (direct comparison by laying one shape on top of the other) and by measuring the sides and angles.

SCO: **SS5: Describe and compare the sides and angles of regular and irregular polygons.**
[C, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Sort a given set of 2-D shapes into polygons and non-polygons, and explain the sorting rule.
- Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by superimposing.
- Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by measuring.
- Demonstrate that the sides of a regular polygon are of the same length and that the angles of a regular polygon are of the same measure.
- Sort a given set of polygons as regular or irregular and justify the sorting.
- Identify and describe regular and irregular polygons in the environment.

SCO: **SS5: Describe and compare the sides and angles of regular and irregular polygons.**
[C, PS, R, V]

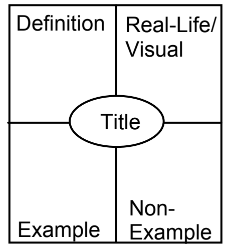
PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

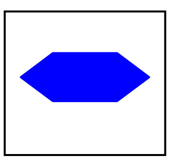


Consider the following strategies when planning lessons:

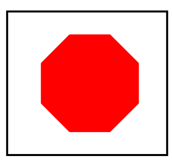
- Provide students with a template for the Frayer Model and have them fill in the sections, individually or as a group, to consolidate their understanding of the properties of polygons and non-polygons. This activity may be repeated to distinguish the attributes of regular and irregular polygons.
- Have students prepare property lists with headings: sides, angles. Using a collection of regular and irregular polygons (models or pictures on cards), have students describe the shapes using language such as: all sides equal, 2 angles the same, opposite sides equal, no sides equal, etc. Then have the students sort the polygons into regular or irregular polygons. Use a Venn diagram or a Carroll diagram to record similarities and differences.
- Provide students with a list of attributes and have them construct a polygon that has the set of attributes. Have students share and compare their polygon with the class.
- Display models or copies of regular polygons on the board. Place a smaller version of the regular polygon on the overhead projector. Have a student move the projector until the image matches, with the one taped on the board. This will help to prove the congruence of their angles, regardless of their side lengths. Interactive white boards can also be an effective tool to show congruency of angles of regular polygons.

Suggested Activities

- Have students work in pairs to prepare a concentration card game with pictures of regular and irregular polygons and their corresponding names.



irregular hexagon



regular octagon

- Have students trace a regular polygon (e.g., yellow pattern block). Have them rotate their shape to prove the congruency of sides and angles. The congruency should be double-checked by measuring the polygon's angles and sides with a protractor and ruler.
- Have students go on a "Polygon and Irregular Polygon" or a "Polygon and Non-Polygon" Scavenger Hunt. Have them sort the polygons they found and explain their rules for sorting.
- Provide several copies of a non-regular polygon that has been rotated and reflected a number of different ways. Have the students cut out one shape and lay it over the others to prove congruency. This can be done using paper drawings or on the computer. Incongruent shapes may be included.
- Have students create various types regular and irregular polygons on geoboards. Have them also create sets of congruent polygons on their geoboards in different orientations. Record on grid paper.

Possible Models: pattern blocks, attribute blocks, tangrams, geoboards, geo-strips, Power Polygons™

SCO: **SS5: Describe and compare the sides and angles of regular and irregular polygons.** [C, PS, R, V]

ASSESSMENT STRATEGIES

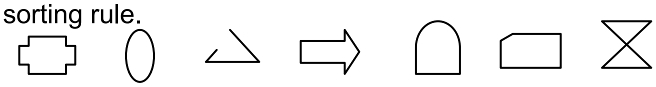
Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

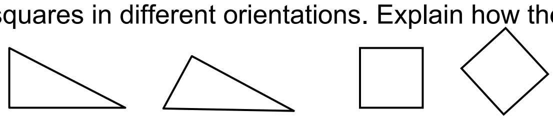
Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

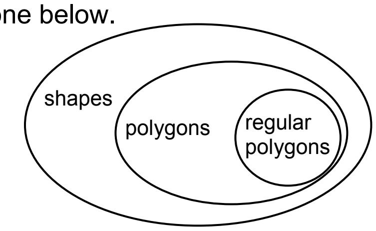
- Provide a set of polygons (paper or other models). Have students determine which are congruent.
- Ask students to draw a polygon and a non-polygon, and explain why one is a polygon and the other is not.
- Provide students with several different polygons (regular and irregular) to sort and have them justify their sorting rule.
- Provide students with several different shapes (polygons and non-polygons) to sort and have them justify their sorting rule.



- Ask students to describe the characteristics of a regular polygon and how they would prove a shape is a regular polygon.
- Provide students with dot paper or a geoboard (11 x 11 pin) and have them draw or create two triangles or squares in different orientations. Explain how they know they are congruent.



- Have students draw congruent polygons that satisfy a given set of attributes. Students should be able to prove the shapes are congruent by measuring.
- Provide two congruent irregular polygons. Have students prove congruency by measuring and labelling the sides and angles.
- Have students sort a set of shapes using a Venn diagram like the one below.



FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SS6: Perform a combination of translation(s), rotation(s) and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image.**
[C, CN, PS, T, V]
SCO: **SS7: Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations.**
[C, CN, T, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
SS7 Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. **SS8** Identify a single transformation, including a translation, rotation, and reflection of 2-D shapes.	**SS6** Perform a combination of translation(s), rotation(s) and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image. **SS7** Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations.	**SS5** Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).

ELABORATION

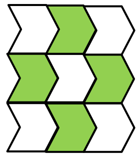
Guiding Questions:
• What do I want my students to learn?
• What do I want my students to understand and be able to do?

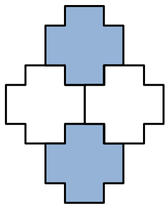
In Grade 5, students learned that there are three transformations that change the location of an object in space, and/or its orientation, but not its size or shape. The three types of transformations are: **translations**, **reflections** and **rotations**. These transformations result in images that are **congruent** with the original object.

- **translations** move a shape left, right, up, down or diagonally without changing its orientation. A real-life example of a translation may be a chess piece moving on a chessboard.
- **reflections** can be thought of as the result of picking up a shape and flipping it over. The reflected image is the mirror image of the original. A real-life example of a reflection may be a pair of shoes.
- **rotations** move a shape around a **turn centre**. A real-life example of a rotation may be clock hands.

In Grade 6, students are expected to perform a combination of successive transformations with 2-D shapes. This could involve a single type of transformation repeated or more than one type of transformation (e.g., reflections and translations). Students will need to be able to describe and model the transformations. It is important for students to recognize that some transformations can be described in more than one way.

Students also must be able to create their own designs using a combination of successive transformations. It is also expected that students are able to analyze existing designs and describe the transformations used to create that design.


SS6


SS7

SCO: **SS6: Perform a combination of translation(s), rotation(s) and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image.**
[C, CN, PS, T, V]
SCO: **SS7: Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations.**
[C, CN, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

SS6
- Demonstrate that a 2-D shape and its transformation image are congruent.
- Model a given set of successive translations, successive rotations or successive reflections of a 2-D shape.
- Model a given combination of two different types of transformations of a 2-D shape.
- Draw and describe a 2-D shape and its image, given a combination of transformations.
- Describe the transformations performed on a 2-D shape to produce a given image.
- Model a given set of successive transformations (translation, rotation, or reflection) of a 2-D shape.
- Perform and record one or more transformations of a 2-D shape that will result in a given image.

SS7
- Analyze a given design created by transforming one or more 2-D shapes, and identify the original shape and the transformations used to create the design.
- Create a design using one or more 2-D shapes and describe the transformations used.
- Model a given set of successive transformations (translation, rotation and/or reflection) of a 2-D shape.

SCO: **SS6: Perform a combination of translation(s), rotation(s) and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image.**
[C, CN, PS, T, V]
SCO: **SS7: Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations.**
[C, CN, T, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students use shapes from the pattern blocks, tangrams, logic blocks, and other sources, to predict and confirm the results of the various transformations.
- Give students pictures of shapes and their images under various transformations. Have them predict what the relationships are and then confirm, using tracing paper or a mira.
- Have students discuss their predictions prior to performing given transformations to a shape.
- Have students investigate such questions as:
  - If a shape undergoes 2 translations, does it matter in which order they take place?
  - Does a reflection followed by a translation produce the same result as the translation followed by the reflection?
- Use wallpaper or fabric as sources of designs which utilize transformational geometry. Students can look at the designs to find evidence of translations, reflections and rotations. Have them record the transformations they observe. Many wallpaper and fabric designs incorporate multiple transformations, and some include interesting tessellations.
- Explore examples of transformations in artists' work such as M.C. Escher (http://www.mcescher.com/).

Suggested Activities

- Provide pattern blocks and have students practice transformations and draw them on grid paper.
- Have students choose a pattern block, perform several transformations of their choice, draw the transformations on grid paper and have a partner describe the transformations that were performed.
- Have students respond in their journal to the following prompts:
- Explain using words and pictures if a translation can ever look like a reflection.
- Explain using words and pictures how you know if a figure and its image show a reflection, translation, or rotation.
- Place three geoboards side by side. Have one student make a scalene triangle on the first geoboard. Ask another student to construct on the second geoboard the image of this triangle if the right side of the first geoboard is used as a mirror line. Ask another student to construct on the third geoboard the image of the triangle on the second geoboard under a 90 degree counterclockwise rotation. Repeat this activity using other shapes and transformations.
- Use technology to demonstrate transformations. This could include websites (e.g., "The National Library of Virtual Manipulatives"), *Geometer's Sketchpad* software, and *Smart Notebook* software.
- Have students choose a 2-D shape and create their own design using a combination of successive transformations. Have students record their transformations, so the design can be reproduced.
- Use a pentomino to perform a combination of transformations, then sketch the pattern on grid paper.

Possible Models: geoboards, grid paper, pattern blocks, tracing paper, Miras, pentominoes, tangrams

SCO: **SS6: Perform a combination of translation(s), rotation(s) and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image.**
[C, CN, PS, T, V]
SCO: **SS7: Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations.**
[C, CN, T, V]

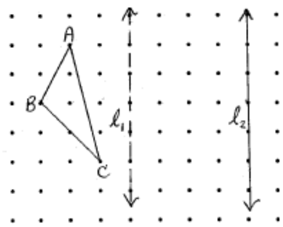
ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

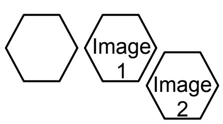
Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment
- Have students prove that a 2-D shape and its transformation image are congruent.
- Have students locate the image of $\triangle ABC$ after a reflection across line 1 followed by a reflection across line 2. Ask them what single transformation of $\triangle ABC$ would have the same result.



- Have students determine which transformations were performed on a given shape.
- Provide students with a 2-D shape and have them follow directions of successive transformations or a combination of transformations.
- Have students explain the transformations shown in a pattern, such as fabric, wallpaper, or other designs.
- Present students with three pictures on grid paper of two congruent shapes after two transformations were performed on them. Ask students to predict what two transformations were performed. Could this have been done in more than one way? Could this have been done by a single transformation?



FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SS8: Identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs.**
[C, CN, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
	SS8 Identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs.	**SS4** Identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

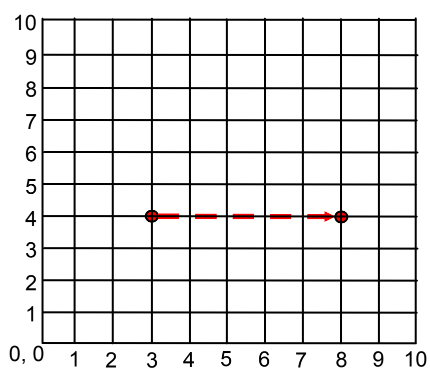
In earlier grades, students have experienced vertical and horizontal number lines. Students will have started to develop an understanding of a coordinate system through graphing activities.

Students need to be able to label the axes of the first **quadrant** of the **Cartesian plane**. They should know that the horizontal axis is the **x-axis** and the vertical is the **y-axis**. Students should extend their knowledge of graphing to determine the location on a Cartesian plane using **coordinates**. Coordinates are written as an **ordered pair** and are written in brackets with a comma to separate the two numbers.

The first number in an ordered pair shows the distance from the **origin (0, 0)**, along the horizontal axis (how far to move to the right). The second number shows the distance from the horizontal axis along a vertical line (how far to move up). Together these numbers are the ordered pair. For example, if we move 3 right and 4 up, the resulting ordered pair is (3, 4). Students need to know to always start at the origin (point where the 2 axes meet).

Students need to be able to determine the distance between points on a grid as well. In Grade 6, students will be expected to figure out distance between two points either horizontally or vertically on the same line. A real-life application of this concept is determining distances between places on a map using the grid lines or a scale.

The distance between point (3, 4) and point (8, 4) is 5 units.



SCO: **SS8: Identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs.**
[C, CN, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Label the axes of the first quadrant of a Cartesian plane and identify the origin.
- Plot a point in the first quadrant of a Cartesian plane given its ordered pair.
- Match points in the first quadrant of a Cartesian plane with their corresponding ordered pair.
- Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole number ordered pairs.
- Draw shapes or designs in the first quadrant of a Cartesian plane, using given ordered pairs.
- Determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane.
- Draw shapes or designs in the first quadrant of a Cartesian plane and identify the points used to produce them.

SCO: **SS8: Identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs.**
[C, CN, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Display a coordinate grid when you introduce these concepts. Be sure to label the axes accurately. A common error students make when labelling the axes is putting the number between the grid lines.
- Draw and label a coordinate grid on the board. Have students explore how they could use two numbers to describe a point on the grid. Introduce terminology such as ordered pair and the origin (0, 0). Also, have students use the words right and up (in this order) as they move along the grid.
- Select points on a grid and have students decide what 2 numbers name these points. If students say the numbers (1, 3) as "three, one" then simply remind them that the first number tells the horizontal distance and the second number tells the vertical distance.
- Play "4 in a row" with the class divided into two teams: "X's or O's ". Each student gets a turn to call out an ordered pair for his/her team. The first team to make 4 in a row wins. This will reinforce the use of the correct order of coordinates.

Suggested Activities

- Ask students to plot 10 points in quadrant 1 for which the difference between the first and second coordinate is 3.
- Give the partial coordinates of a square (1, 2), (1, 7), (6, 2). Ask students to find the last point and label the coordinates.
- Give students a grid with 5 points on it and have them match these to 5 ordered pairs listed below it.
- Have students plot points on grids with different scales (e.g., intervals of ones, twos, fives, tens).
- Create "join-the-dots" pictures on a coordinate grid to reinforce locating coordinates. After they draw their pictures on a grid, they list the coordinates in order of connection. The list of coordinates can be given to other students who then use them to recreate the picture.
- Play "battleship" on the first quadrant of the Cartesian plane. Each player will need two copies of the grid; one grid to mark his/her ships on and one grid to keep track of the ordered pairs he/she is asking and whether or not it was a hit or miss.
- Create a picture as a class with labelled points and then transform the picture by increasing the x and y values. For example, a new figure could be made by changing each point using (x+5, 2y).

Possible Models: grid paper, ruler, maps

SCO: **SS8: Identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs.**
[C, CN, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Give students a street map with grid lines on it. Have them determine the distance between locations on this map.
- Tell students that a town map is drawn on a grid. The fire station is at (40, 30). There are four fire hydrants, each 20 units from the station in a straight horizontal or vertical line. Have them draw and label axes on the grid, explaining what scale they used, plot the fire station, list the ordered pairs where the hydrants would be found and plot the points.
- Ask students to explain how to use ordered pairs to describe and locate points on a grid.
- Have students predict the shape that was created by plotting points and joining them with straight lines for the following coordinates: (3, 0), (4, 0), (5, 2), (4, 5), (3, 4), (2, 2). Have students then create the shape.
- Tell students that two objects were placed at (0, 4) and (3, 7) on a grid. Have them describe where the second object was placed in relation to the first. Then plot the points and check.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SS9: Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).** [C, CN, PS, T, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

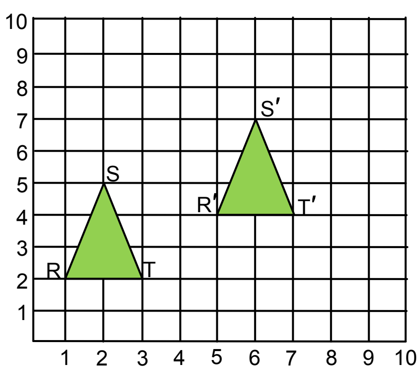
Grade Five	Grade Six	Grade Seven
SS7 Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. **SS8** Identify a single transformation, including a translation, rotation, and reflection of 2-D shapes.	**SS9** Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).	**SS5** Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).

ELABORATION

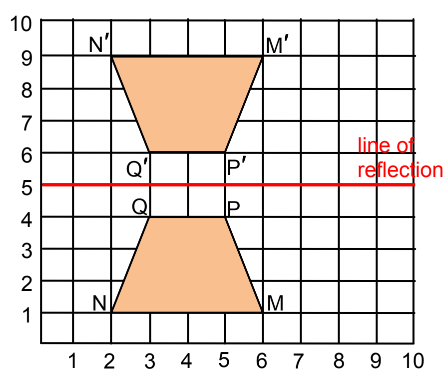
Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In Grade 5, students learned there are three transformations that change the location of an object in space, or the direction in which it faces, but not its size or shape. These are: **translations**, **reflections**, and **rotations**. These transformations are further explored in Grade 6 in outcomes SS6 and SS7. Students will also require knowledge of plotting coordinates on a Cartesian plane as described in SS8.

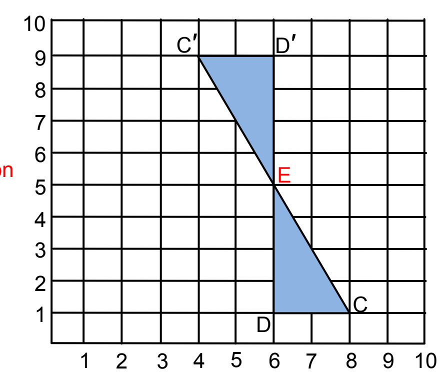
Students are expected to identify and perform these three types of transformations on a Cartesian plane, identify the coordinates of the new **image** (A′B′C′D′: read as A prime, B prime, C prime and D prime) and describe the change (e.g., when the image below was translated, each *x* coordinate increased by 4 because the shape was translated 4 units to the right). Examples of each of these are shown below.



Translation
(4 right, 2 up)



Reflection
(line of reflection)



Rotation
(180° clockwise around vertex E)

For this outcome, students are only expected to perform a **single transformation** in the first quadrant.

SCO: **SS9: Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).**
[C, CN, PS, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify the coordinates of the vertices of a given 2-D shape (limited to the first quadrant of a Cartesian plane).
- Perform a transformation on a given 2-D shape and identify the coordinates of the vertices of the image (limited to the first quadrant).
- Describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to first quadrant).

SCO: **SS9: Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).**
[C, CN, PS, T, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students **many** opportunities to *translate* a given 2-D shape on a Cartesian plane on grid paper, identify the coordinates of the vertices and describe the positional change of the vertices.
- Provide students **many** opportunities to *rotate* a given 2-D shape on a Cartesian plane on grid paper, identify the coordinates of the vertices and describe the positional change of the vertices. Students may trace the original shape on paper (wax or tracing) and use the point of their pencil pressed down on the point of rotation to help them rotate the shape.
- Provide students **many** opportunities to *reflect* a given 2-D shape on a Cartesian plane on grid paper, identify the coordinates of the vertices and describe the positional change of the vertices. Students may use Miras on the given line of reflection. Include opportunities where the line of reflection is horizontal, vertical, and diagonal.
- Have students discuss their predictions prior to performing a given transformation to a shape.
- Explore this concept in other curricular areas such as art and physical education.
- Provide shapes cut from cardstock that have vertices that fit on 1cm grid paper. Have them practice performing, drawing, and recording various transformations.

Suggested Activities

- Have students describe the direction as well as the size/magnitude of a given translation.
- Have students determine which transformation was performed on a given shape.
- Provide pattern blocks and have students practice each transformation and draw them on a Cartesian plane on grid paper.
- Have students choose a pattern block, perform a transformation of their choice, draw the transformation on a Cartesian plane on grid paper and have a partner describe the transformation that was performed, including the coordinates of the original vertices and the new image's vertices.
- Ask students to complete:
  - a rotation given the direction of the turn (clockwise or counter-clockwise), the degree or fraction of the turn (e.g., 90°, three quarter) and the point of rotation;
  - a translation given the direction and size/magnitude of the movement;
  - a reflection given the line of reflection and the distance from the line of reflection, limited to remaining in the first quadrant.
- Ask students to create a shape on the geoboard, perform a transformation of their choice, and describe the transformation that was performed. Then repeat this on a grid (limited to the 1st quadrant).
- Respond in their journal to the following prompts:
  - Explain using words and pictures if a translation can ever look like a reflection.
  - Explain using words and pictures how you know if a figure and its image shows a reflection, translation, or rotation.

Possible Models: geoboards, grid paper, pattern blocks, Miras

SCO: **SS9: Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).**
[C, CN, PS, T, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment
- Provide students with diagrams of different transformations and have them label each diagram with the type of transformation the diagram shows, including the coordinates of the vertices of both images.
- Have students draw a shape, translate it, and then describe the positional change of the vertices.
- Ask students to describe how the translation rule can help them identify the coordinates of the vertices of the new image.
- Provide a 2-D shape and have students perform a rotation, reflection, or translation on grid paper of that shape, label and identify the coordinates of the vertices of both images and describe the positional change.
- Ask students to explain the differences and similarities among the three different transformations with regard to the Cartesian plane and the coordinates of the figure and its image.
- Explain using words and pictures how you know if a figure and its image show a reflection, translation, or rotation.
- Provide the coordinates for a shape and its transformation. Have the students plot and draw both shapes and describe the transformation that has occurred.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SP1: Create, label and interpret line graphs to draw conclusions.** [C, CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
SP2 Construct and interpret double bar graphs to draw conclusions.	**SP1** Create, label and interpret line graphs to draw conclusions.	**SP3** Construct, label and interpret circle graphs to solve problems.

ELABORATION

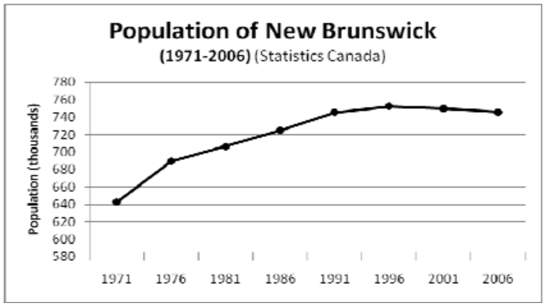
Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students have investigated tables of values and describing patterns and relationships using graphs and tables in outcomes PR1 and PR2.

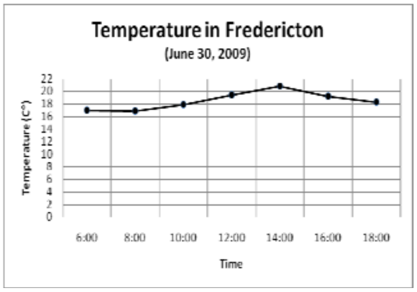
The points on a line graph are plotted to show relationships between two variables. Points are then joined to form a line to make it easier to focus on trends implicit in the data. Line graphs should include a title, labelled axes (overall description and specific data categories), and a clear scale. Line graphs are not always straight lines and may be called broken-line graphs.

Every point on the line should have a value, but a line graph can also be used to show values between points on the graph. Students should be able to determine the value of the data points.



The purpose of a line graph is to focus on trends implicit in the data. For example, if students measure the temperature outside every hour during a school day, they could create a graph in which the ordered pairs (hour, temperature) are plotted. By connecting the points with line segments, they see the trend in the temperature. This type of exploration of line graphs links to outcome SS8. Ensure that the construction of the line graph and interpretation of the data are not addressed independently. Whenever students construct graphs, the data should be discussed and interpreted.

The distinction between **continuous** and **discrete** data should be emphasized as students investigate line graphs. Continuous data includes an infinite number of values between two points and is shown by joining the data points. Discrete data has finite values (i.e., data that can be counted such as the number of pets), and the data between the points have no value. As a result the points in the graph should not be connected and no inferences can be made about values between two data points.



SCO: **SP1: Create, label and interpret line graphs to draw conclusions.**
[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Determine the common attributes (title, axes and intervals) of line graphs by comparing a given set of line graphs.
- Determine whether a given set of data can be represented by a line graph (continuous data) or a series of points (discrete data) and explain why.
- Create a line graph from a given table of values or set of data.
- Interpret a given line graph to draw conclusions.

SCO: **SP1: Create, label and interpret line graphs to draw conclusions.**
[C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Ensure students are aware of the parts of line graphs (e.g., titles, labels, scales, etc.) using real graphs that are interesting to students.
- Have students create line graphs and have them explain and justify the attributes they used when they created their graphs (scales, labels, title, etc.).
- Connect this outcome with prior knowledge of tables of values or sets of data (PR1 and PR2.
- Provide real-world line graphs and ask questions that require students to read and interpret the information found there.
- Have a whole group discussion about the differences between continuous and discrete data and when to use each type of graph.
- Integrate the use of technology to construct line graphs. It is important that students also have the experience of creating graphs with paper-and-pencil methods.
- Use websites such as Statistics Canada (http://www.statcan.gc.ca/) which include background information, activities, and lesson plans (http://www.statcan.gc.ca/kits-trousses/courses-cours/edu05_0017-eng.htm#link08).

Suggested Activities

- Have students collect information about the number of students in the school in Grades 1, 2, 3, 4, and 5 and draw a line graph to help show whether there are differences in the number of students in certain grades. Remind students to carefully consider the step size for the vertical scale.
- Have students record the changes in temperature over time during the day/week and create an appropriate line graph and label the title, axes, and scales.
- Ask students to look up the hockey scores for a favourite team over the course of 10 games and then create a line graph with the ordered pairs (game number, number of goals scored by favourite team). Have them create a second graph with the ordered pairs (game number, goals scored by opposing team) and then compare the two graphs.
- Have a class discussion about the differences between continuous and discrete data and when to use each type of graph.

Possible Models: graph paper, computer programs (spreadsheet or graphing applications)

SCO: **SP1: Create, label and interpret line graphs to draw conclusions.**
[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Provide students with two line graphs displaying similar data (such as temperature change over time in two different areas) and have students write comparison statements based on the data shown.
- Have students create a line graph based on the following information using appropriate scales, labels, and title.

Number of Cups	1	2	3	4
Capacity (mL)	250	500	750	1000

- Have students explain (in words or pictures) the difference between continuous and discrete data.
- Provide an example of a line graph and have students create three questions which can be answered from the graph.
- Have students explain three situations where it would be appropriate to use a line graph.
- Provide a broken-line graph and have students explain why line graphs are not always linear.
- Provide students with examples of different types of data. Have them determine whether the data is continuous or discrete.
  - The number of students who eat in the cafeteria over a month.
  - The temperature over 48 hours.
  - The attendance at the local movie theatre.
  - Your height over 5 years.
- Have students create a line graph based on the table below. Have them determine about how much rain fell by 5:30 p.m. If the rain continues to fall steadily at the same rate, about how much will fall by 8:00 p.m.?

Time of Day	2:00 p.m.	3:00 p.m.	4:00 p.m.	5:00 p.m.	6:00 p.m.
Total rainfall	3 mm	5 mm	7 mm	9 mm	11 mm

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SP2: Select, justify and use appropriate methods of collecting data, including:** • **questionnaires** • **experiments** • **databases** • **electronic media.** [C, PS, T]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
SP1 Differentiate between first-hand and second-hand data.	**SP2** Select, justify and use appropriate methods of collecting data, including: questionnaires; experiments; databases; electronic media.	**SP3** Construct, label and interpret circle graphs to solve problems.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

From Grade 5, students should recognize that although some data is collected **first-hand** by interviews or observations, much of the data to which they are exposed is **second-hand**. Students should explore, through discussion, how such data might be collected and how reliable they feel it is. For example, if students read that 30% of children in Canada are not physically fit, what might they wonder about the data source? Was a sample used? Were children tested directly or was data collected by asking doctors or teachers? Students should realize that they must be careful about drawing conclusions from reported data. Becoming familiar with sources for different types of data is valuable to students.

There are many different sources of data. A questionnaire is a collection of survey questions on the same topic. When designing a questionnaire, it is important to formulate good questions. It is helpful to discuss with students the various options in designing their questionnaire (e.g., multiple choice responses or yes/no responses; interviews or independently completed questionnaires).

Data can also be collected by conducting an experiment that is set up to answer a particular question. Electronic media, such as spreadsheets or Internet sites (e.g., Statistics Canada, music databases, Guinness World Records, weather, or sport leagues) are another useful source of data.

If information about a large population is needed, there are organized collections of related data called databases, such as those created by Statistics Canada. Sometimes it is not possible to survey every person. In these situations a **sample** of the population is used and the results are then generalized to the entire target group. When this data is analyzed, it is important that students recognize that conclusions drawn from the sample may not be perfectly true for the entire group. A sample should also be carefully selected to avoid potential biases. For example, if someone wanted to determine the favourite type take-out food in a community, it would not be reliable to only survey patrons of “Pizza King”. This sample of people would likely be biased in favour of pizza. After students have collected their data, have students explore which type of graph(s) would be appropriate to display it.

SCO: **SP2: Select, justify and use appropriate methods of collecting data, including:**
- **questionnaires**
- **experiments**
- **databases**
- **electronic media.**

[C, PS, T]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Select a method for collecting data to answer a given question and justify the choice.
- Design and administer a questionnaire for collecting data to answer a given question, and record the results.
- Answer a given question by performing an experiment, recording the results and drawing a conclusion.
- Explain when it is appropriate to use a database as a source of data.
- Gather data for a given question by using electronic media including selecting data from databases.

SCO: **SP2: Select, justify and use appropriate methods of collecting data, including:**
- **questionnaires**
- **experiments**
- **databases**
- **electronic media.**

[C, PS, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students collect data to solve problems that are relevant to them. Begin by having students choose good survey questions with a limited number of answers (possibly "Other" as one choice). The response options should be discrete and not overlap.
- Have students design questionnaires with the audience and situation in mind. Students should be made aware that many factors have the potential to affect the results, including bias and sample size.
- Remind students that data can be first-hand (collected directly by students) or second-hand (collected by others).
- Use websites such as Statistics Canada (http://www.statcan.gc.ca/kits-trousses/cyb-adc2001/edu04_0035e-eng.htm) as a source of data and additional information on statistics and various data displays.

Suggested Activities

- Have students find the answer to "Which hockey player scored the most goals in one season?" using Internet/electronic media databases.
- Have students design and conduct experiments to answer a question. For example, an experiment could be on memory where 20 items are viewed for one minute, then covered, and the subject has to name as many as they can.
- Have students, working in pairs, design a questionnaire for a given question, administer it, and record the results.
- Ask students what sample/data source they would use to answer questions such as the amount of water an average Canadian uses in a day.
- Have students design a questionnaire on problems such as: "What nutritious snacks should be placed in our vending machines?" or "How many hours do Grade 6 students spend using the Internet each day?" Have students collect the data and later graph the results (SP3).

Possible Models: grid paper, computer programs (spreadsheet or graphing applications)

SCO: **SP2: Select, justify and use appropriate methods of collecting data, including:**
- **questionnaires**
- **experiments**
- **databases**
- **electronic media.**

[C, PS, T]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment
- Ask students why a sample of 5-year-olds might not be the best one to find out what playground equipment an elementary school should have.
- Have students describe how they would gather data on the following and justify their method.
  - the three most popular selections out of their school vending machine;
  - the daily high temperature for Halifax for the past three weeks;
  - the number of times "heads" turns up out of 100 flips of a coin.
- Provide two sample survey questions. Ask students which is better. Have students give a reason for their choice.
  a. How many brothers and sisters do you have? ___
  b. Are you a member of a large family? Yes ___ No ___
- Have students, in pairs, design a questionnaire for a given question, administer it, and record the results.
- Ask students to create a graph that shows the growth of the population of New Brunswick over a period of 20 years. What databases could be accessed to find the information?
- Have students use a spinner and record the results. Ask if they are able use the results to draw a conclusion on the following question: What is the favourite color of students in Grade 6? Explain.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SP3: Graph collected data and analyze the graph to solve problems.** [C, CN, PS]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
SP2 Construct and interpret double bar graphs to draw conclusions.	**SP3** Graph collected data and analyze the graph to solve problems.	**SP3** Construct, label and interpret circle graphs to solve problems.

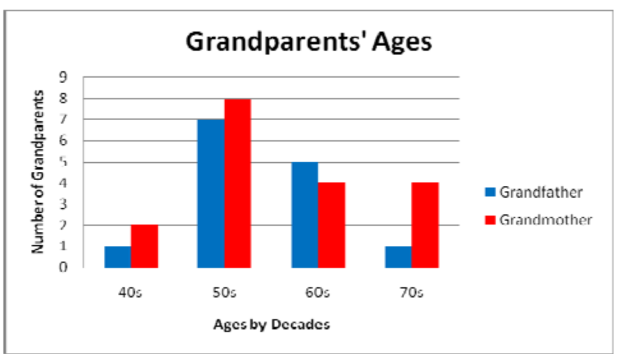
ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students should regularly use a variety of graphs to display and organize data. Discuss the different types of graphs that students know and how these are used to display different types of information. By the end of Grade 6, students should know how to create and analyze pictographs, line plots, Venn diagrams, Carroll diagrams, bar graphs, double bar graphs, and line graphs. Students will study circle graphs in Grade 7, but they may already be aware of these and could be part of this discussion.

As described in SP2, data can be collected in surveys, through experiments, or through research. Topics may include areas of mathematics, other curricular areas such as science and social studies and in real-life situations. For example, students might gather information about the ages of their grandparents and display it in various types of graphs.



Grandparents' Ages

Grandparents		
In their 40s:	Grandfather Grandmother	☺ ☺ ☺
In their 50s:	Grandfather Grandmother	☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺
In their 60s:	Grandfather Grandmother	☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺
In their 70s:	Grandfather Grandmother	☺ ☺ ☺ ☺ ☺
☺ = 1 grandparent		

After students have collected their data, they should be able to justify which type of graph(s) would be appropriate to use to display it. Students should recognize that the various types of data displays are not always equally effective or appropriate depending on the type of data. For example, students should recognize that a line graph would not be appropriate for the information displayed in the graphs above since the data is a count of the number of grandparents in each age range and is therefore not continuous. When students create graphs, ensure that they include a title, labels on both axes, and an appropriate scale. Data displays communicate information, so it is important that graphs are accurate, well-organized, and easy to read.

Students need to understand that data is collected to answer questions and solve problems. "When students formulate the questions they want to ask, the data they gather become more and more meaningful. How they organize the data and the techniques for analyzing them have a purpose" (Van de Walle & Lovin, vol. 3, 2006, p. 309).

SCO: **SP3: Graph collected data and analyze the graph to solve problems.**
[C, CN, PS]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Determine an appropriate type of graph for displaying a set of collected data and justify the choice of graph.
- Solve a given problem by graphing data and interpreting the resulting graph.

SCO: **SP3: Graph collected data and analyze the graph to solve problems.**
[C, CN, PS]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Examine many real-world pictographs, bar, double bar, and line graphs gathered from newspapers, magazines, and other print media. Discuss why the choice of format is appropriate in each case. Ask students questions that can be answered through careful analysis of the graph.
- Collect data as a class or individually. Have students place the data in a table, and choose an appropriate graph to display it. Ask students to explain their reasoning for their choice of graph.
- Use the Internet as a source of data and possible lesson ideas such as:
  - Statistics Canada Statistics Canada (www.statscan.ca)
  - NBED Portal Research tools: World Book online, National Film Board, and more (https://portal.nbed.nb.ca/tr/rt/Pages/default.aspx) (log in and password information on page)
- Provide meaningful questions that students can answer by gathering and graphing data. Examples:
  - If we order t-shirts for our school, what are the most popular sizes we need to get?
  - What were the most frequently observed types of insects during our science investigation?
  - What are the distances our paper airplanes travelled in our “Flight” experiment?
  - What type of fruit was purchased the most at the school canteen or cafeteria?
- Provide students with questions to help them to analyze the data.
  - Which data point is the greatest? Least? Why do you think this is the case?
  - What trend does the data show?
  - What predictions can you make?
  - What questions do you have based on the graph?

Suggested Activities

- Give groups of students examples of different types of graphs. Have them create reasons for when and why we would use this type of graph. Combine ideas and have students present their findings. Students could also create a list of questions relating to the graph that could then be analyzed.
- Explore ideas as classroom questions to collect data, graph, and analyze such as:
  - Favourites: types of music, sports, video games, movies;
  - Numbers: amount of money spent on entertainment (movies, etc.), number of pets, hours on computer, number of texts per week;
  - Measures: sitting height, arm span, area of foot, time on the bus.

  (Van de Walle & Lovin, vol. 3, 2006, p. 309).
- Present students with “real life” survey questions such as students’ satisfaction with cafeteria food, the most popular noon hour activity, or whether students would like to have a school uniform. Have them collect the data, display it with an appropriate graph and interpret the results.
- Have students explore how data is displayed in other subject areas or in the media. Discuss how the graphs can be analyzed to solve problems.

Possible Models: grid paper, computer programs (spreadsheet or graphing applications), prepared graphs from media such as newspapers or magazines

SCO: **SP3: Graph collected data and analyze the graph to solve problems.**
[C, CN, PS]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask students to describe the purpose of different types of graphs and give examples of types of data that are appropriate and inappropriate for each (e.g., pictograph, bar graph, line graph).
- Ask students to create a graph that compares two sets of data. Have them explain their choice of graph. Ensure that students include a title, labels on both axes, an appropriate scale, and is well-organized.
- Have students answer a given question by performing an experiment or collecting data. Students should record the results, graph the data, and draw conclusions based on the data and graph.
- Provide students with a collection of data and have students graph the information. Consider the student's choice of graph format and the presence of title, labels, appropriate scales, and accurate data representation.
- Provide students with a graph. Ask them to describe what they can interpret from it. Have them graph the same data using a different type of data display.
- Provide students with a graph and have them answer questions that require careful analysis of the data shown.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SP4: Demonstrate an understanding of probability by:**
- **identifying all possible outcomes of a probability experiment**
- **differentiating between experimental and theoretical probability**
- **determining the theoretical probability of outcomes in a probability experiment**
- **determining the experimental probability of outcomes in a probability experiment**
- **comparing experimental results with the theoretical probability for an experiment.**

[C, ME, PS, T]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Five	Grade Six	Grade Seven
SP3 Describe the likelihood of a single outcome occurring using words, such as: impossible**;** possible**;** certain. **SP4** Compare the likelihood of two possible outcomes occurring using words, such as: less likely**;** equally likely; more likely.	**SP4** Demonstrate an understanding of probability by: • identifying all possible outcomes of a probability experiment • differentiating between experimental and theoretical probability • determining the theoretical probability of outcomes in a probability experiment • determining the experimental probability of outcomes in a probability experiment • comparing experimental results with the theoretical probability for an experiment.	**SP4** Express probabilities as ratios, fractions and percents.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students were introduced to the concept of **probability** in Grade 5. Probability is a measure of how likely an event is to occur. Probability is about predictions of events over the long term rather than predictions of individual, isolated events. **Theoretical probability** can sometimes be obtained by carefully considering the possible outcomes and using the rules of probability. For example, in flipping a coin, there are only two possible outcomes, so the probability of flipping a head is, in theory, $\frac{1}{2}$. Often in real-life situations involving probability, it is not possible to determine theoretical probability. We must rely on observation of several **trials** (experiments) and a good estimate, which can often be made through a data collection process. This is called **experimental probability**.

Theoretical probability of an event is the ratio of the number of favourable outcomes in an event to the total number of possible outcomes, when all possible outcomes are equally likely. Simply stated, theoretical probability describes what "should" happen and helps predict the experimental probability.

$$\text{Theoretical probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

Experimental probability, or the relative frequency of an event, is the ratio of the number of observed successful occurrences of the event to the total number of **trials**. The greater the number of trials, the closer the experimental probability approaches the theoretical probability. Before conducting experiments, students should predict the probability whenever possible.

$$\text{Experimental probability} = \frac{\text{Number of observed successful occurrences}}{\text{Total number of trials in the experiment}}$$

SCO: **SP4: Demonstrate an understanding of probability by:**
- **identifying all possible outcomes of a probability experiment**
- **differentiating between experimental and theoretical probability**
- **determining the theoretical probability of outcomes in a probability experiment**
- **determining the experimental probability of outcomes in a probability experiment**
- **comparing experimental results with the theoretical probability for an experiment.**

[C, ME, PS, T]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- List the possible outcomes of a probability experiment, such as:
  - tossing a coin
  - rolling a die with a given number of sides
  - spinning a spinner with a given number of sectors.
- Determine the theoretical probability of an outcome occurring for a given probability experiment.
- Predict the probability of a given outcome occurring for a given probability experiment by using theoretical probability.
- Conduct a probability experiment, with or without technology, and compare the experimental results to the theoretical probability.
- Explain that as the number of trials in a probability experiment increases, the experimental probability approaches theoretical probability of a particular outcome.
- Distinguish between theoretical probability and experimental probability, and explain the differences.

SCO: **SP4: Demonstrate an understanding of probability by:**
- **identifying all possible outcomes of a probability experiment**
- **differentiating between experimental and theoretical probability**
- **determining the theoretical probability of outcomes in a probability experiment**
- **determining the experimental probability of outcomes in a probability experiment**
- **comparing experimental results with the theoretical probability for an experiment.**

[C, ME, PS, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

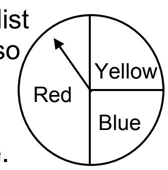
Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Introduce simulations: experiments which indirectly model a situation. Students will have had experience directly determining experimental probabilities in Grade 5. An example of a simulation is creating a spinner that represents a basketball player who makes their free throws 8 times in 10. The spinner has 0.8 of the face labelled HIT and 0.2 labelled MISS. This can also be simulated with a 10-sided dice: the numbers 1 to 8 representing HITS and numbers 9 and 10 representing MISSES. Either model can be used to simulate:
  - the probability of making exactly 3 shots in the next 5 tries;
  - the probability of missing the first shot, but making the next 3 in a row;
  - the probability of missing 5 shots in a row.
- Have students explore situations for which outcomes are equally likely. In these cases, they should list the outcomes and count the number of items on the list to determine probabilities. Students must also recognize, however, when outcomes are not equally likely and take this into account. For example, using the spinner shown, students might list the outcomes as “red,” “yellow” and “blue” and assume that since there are 3 outcomes, each has a probability of $\frac{1}{3}$. This, however, is not the case.



  Students might benefit from reconfiguring the spinner to show equally likely outcomes by dividing the red section into two equal pieces. Now the outcomes might be “red 1,” “red 2,” “yellow” and “blue” and each outcome would now have a probability of $\frac{1}{4}$. Because there are two red sections, the probability of red is, therefore $\frac{2}{4}$.

Suggested Activities

- Provide pairs of students with 24 linking cubes of different colours and a paper bag. Have them determine the theoretical probability for selecting each colour from the bag. Next have them conduct the experiment by drawing and replacing one cube at a time for 50 trials. Compare the theoretical and experimental probabilities and discuss.
- Have students determine approximately how many boxes of cereal will need to be purchased before a consumer collects each of six possible prizes contained therein. This simulation can be performed by rolling a die, recording the prize number won (based on the roll of the die), continuing until at least one of each number is rolled, repeating the experiment several times and determining, on average, the number of rolls (purchases) required.
- Have students discuss how probability is used in the media. Ask students to find examples of how probability is used to influence people in advertisements, the Internet, newspapers, and magazines.

Possible Models: dice, spinners, linking cubes, cards, coins

SCO: **SP4: Demonstrate an understanding of probability by:**
- **identifying all possible outcomes of a probability experiment**
- **differentiating between experimental and theoretical probability**
- **determining the theoretical probability of outcomes in a probability experiment**
- **determining the experimental probability of outcomes in a probability experiment**
- **comparing experimental results with the theoretical probability for an experiment.**

[C, ME, PS, T]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

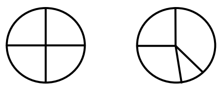
Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask students to create a spinner for which there are 4 equally likely outcomes and another spinner for which the 4 outcomes are not equally likely. Have them predict the probability for each spinner's outcomes.



- Provide students with a bag with 10 red cubes and 5 blue cubes. Ask students to determine the theoretical probability of picking a blue cube.
- Ask students to list the outcomes that result when two dice are rolled and the numbers are subtracted. Conduct the experiment 6 times and compare theoretical and experimental probabilities. Then conduct the experiment 60 times and compare the results with the first results. Explain what happens when you increase the number of trials in a probability experiment.
- Provide students with a 10-sided die and have them determine the theoretical probability of rolling a prime number (2, 3, 5, 7). Have students roll the die 5 times, 10 times, and 50 times and compare the experimental probability result of each with the theoretical probability. Ask them to explain why it is important to have more than a few trials in a probability experiment.
- Tell students that you rolled a pair of number cubes 25 times and the sum of the numbers was 8 on 4 of the rolls. What is the theoretical and experimental probability that the sum is 8?
- Have students explain how a scientific experiment is like a probability experiment. They should focus on the differences between theory/hypothesis and experimental results.

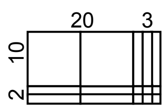
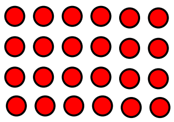
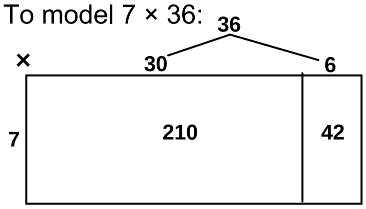
FOLLOW-UP ON ASSESSMENT

Guiding Questions

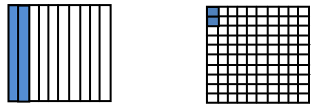
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

GLOSSARY OF MODELS

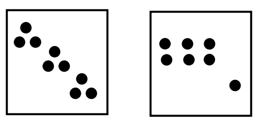
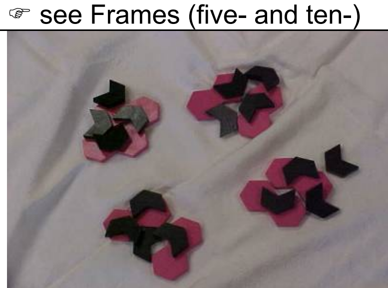
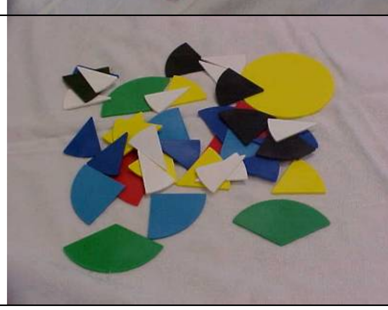
This glossary is identical for all grade levels (kindergarten to grade 9). Most of the models have a variety of uses at different grade levels. More information as to which models can be used to develop specific curriculum outcomes is located on the *Instructional Strategies* section of each four-page spread in this curriculum document. The purpose of this glossary is to provide a visual of each model and a brief description of it.

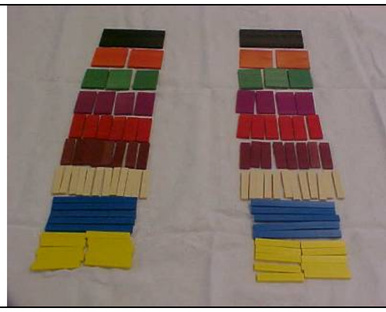
Name	Picture	Description
Algebra tiles		• Sets include "X" tiles (rectangles), "X^2" tiles (large squares), and integer tiles (small squares). • All tiles have a different colour on each side to represent positive and negative. Typically the "X" tiles are green and white and the smaller squares are red and white. • Some sets also include "Y" sets of tiles which are a different colour and size than the "X" tiles.
Area Model	To model 12 × 23: 	• Use base ten blocks to represent the parts of each number that is being multiplied. • To find the answer for the example shown, students can add the various parts of the model: 200 + 30 + 40 + 6 = 276. • This model can also be used for fraction multiplication.
Arrays and Open Arrays	To model 4 × 6:  To model 7 × 36: 	• Use counters arranged in equal rows or columns or a Black line Master with rows and columns of dots. • Helpful in developing understanding of multiplication facts. • Grids can also be used to model arrays. • Open arrays allows students to think in amounts that are comfortable for them and does not lock them into thinking using a specific amount. These arrays help visualize repeated addition and partitioning and ultimately using the distributive property.
Attribute Blocks		• Sets of blocks that vary in their attributes: o 5 shapes circle, triangle, square, hexagon, rectangle o 2 thicknesses o 2 sizes o 3 colours
Balance (pan or beam) scales		• Available in a variety of styles and precision. • Pan balances have a pan or platform on each side to compare two unknown amounts or represent equality. Weights can be used on one side to measure in standard units. • Beam balances have parallel beams with a piece that is moved on each beam to determine the mass of the object on the scale. Offer greater accuracy than a pan balance.

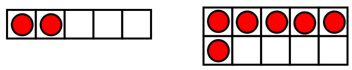
<table>
<tr><td>Base Ten Blocks</td><td></td><td>Include unit cubes, rods, flats, and large cubes.
Available in a variety of colours and materials (plastic, wood, foam).
Usually 3-D.</td></tr>
<tr><td>Beam Balance</td><td colspan="2">☞ see Balance (pan or beam)</td></tr>
<tr><td>Carroll Diagram</td><td>Example:<table><tr><td></td><td>1-digit</td><td>2-digit</td></tr><tr><td>Even</td><td>2, 4, 6, 8</td><td>26, 34</td></tr><tr><td>Odd</td><td>1, 3, 5, 7, 9</td><td>15, 21</td></tr></table></td><td>Used for classification of different attributes.
The table shows the four possible combinations for the two attributes.
Similar to a Venn Diagram</td></tr>
<tr><td>Colour Tiles</td><td></td><td>Square tiles in 4 colours (red, yellow, green, blue).
Available in a variety of materials (plastic, wood, foam).</td></tr>
<tr><td>Counters (two colour)</td><td></td><td>Counters have a different colour on each side.
Available in a variety of colour combinations, but usually are red & white or red & yellow.
Available in different shapes (circles, squares, bean).</td></tr>
<tr><td>Cubes (Linking)</td><td></td><td>Set of interlocking 2 cm cubes.
Most connect on all sides.
Available in a wide variety of colours (usually 10 colours in each set).
Brand names include: Multilink, Hex-a-Link, Cube-A-Link.
Some types only connect on two sides (brand name example: Unifix).</td></tr>
<tr><td>Cuisenaire Rods®</td><td></td><td>Set includes 10 different colours of rods.
Each colour represents a different length and can represent different number values or units of measurement.
Usual set includes 74 rods (22 white, 12 red, 10 light green, 6 purple, 4 yellow, 4 dark green, 4 black, 4 brown, 4 blue, 4 orange).
Available in plastic or wood.</td></tr>
</table>

Decimal Squares®		• Tenths and hundredths grids that are manufactured with parts of the grids shaded. • Can substitute a Black line Master and create your own class set.

Dice (Number Cubes)		• Standard type is a cube with numbers or dots from 1 to 6 (number cubes).
		• Cubes can have different symbols or words.
		• Also available in:
		  o 4-sided (tetrahedral dice)
		  o 8-sided (octahedral dice)
		  o 10-sided (decahedra dice)
		  o 12-sided, 20-sided, and higher
		  o Place value dice

Dominoes		• Rectangular tiles divided in two-halves. • Each half shows a number of dots: 0 to 6 or 0 to 9. • Sets include tiles with all the possible number combinations for that set. • Double-six sets include 28 dominoes. • Double-nine sets include 56 dominoes.
Dot Cards		• Sets of cards that display different number of dots (1 to 10) in a variety of arrangements. • Available as free Black line Master online on the "Teaching Student-Centered Mathematics K-3" website (BLM 3-8).
Double Number Line	☞ see Number lines (standard, open, and double)	
Five-frames	☞ see Frames (five- and ten-)	
Fraction Blocks		• Also known as Fraction Pattern blocks. • 4 types available: pink "double hexagon", black chevron, brown trapezoid, and purple triangle. • Use with basic pattern blocks to help study a wider range of denominators and fraction computation.
Fraction Circles		• Sets can include these fraction pieces: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$ • Each fraction graduation has its own colour. • It is helpful to use ones without the fractions marked on the pieces for greater flexibility (using different piece to represent 1 whole).

<table>
<tr><td>Fraction Pieces</td><td></td><td>

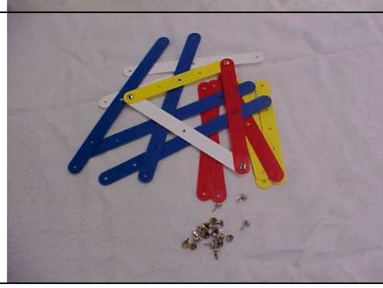
Rectangular pieces that can be used to represent the following fractions:
$\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$
Offers more flexibility as different pieces can be used to represent 1 whole.
Each fraction graduation has its own colour.
Sets available in different quantities of pieces.
</td></tr>
<tr><td>Frames
(five- and ten-)</td><td></td><td>

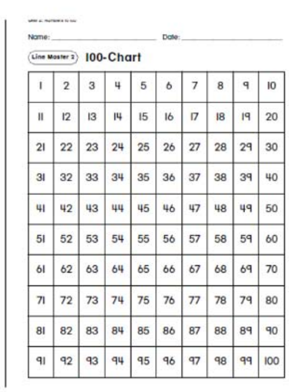
Available as a Black line Master in many resources or you can create your own.
Use with any type of counter to fill in the frame as needed.
</td></tr>
<tr><td>Geoboards</td><td></td><td>

Available in a variety of sizes and styles.

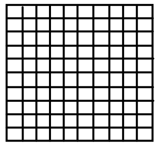
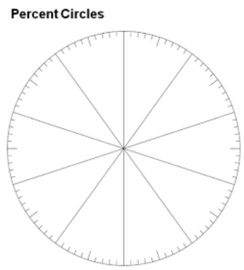
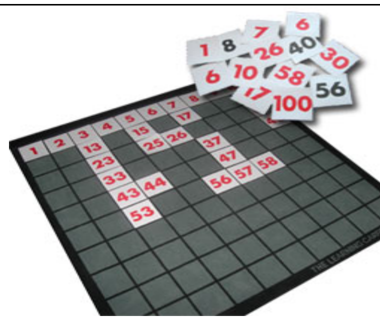
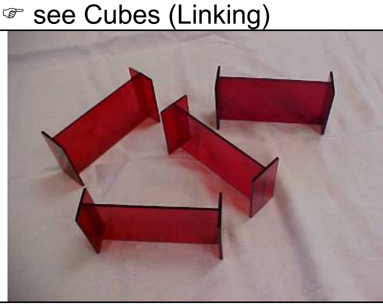
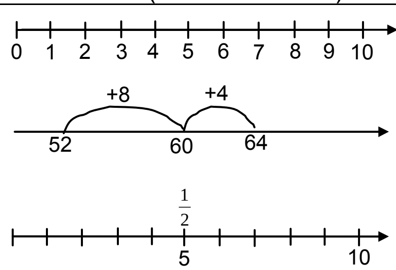
5 × 5 pins
11 × 11 pins
Circular 24 pin
Isometric

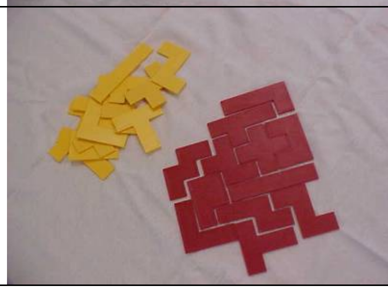
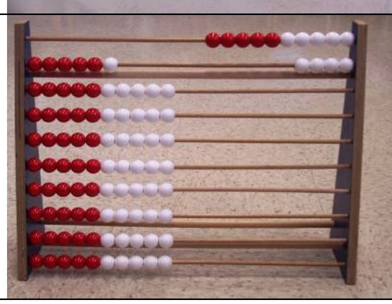
Clear plastic models can be used by teachers and students on an overhead.
Some models can be linked to increase the size of the grid.
</td></tr>
<tr><td>Geometric Solids</td><td></td><td>

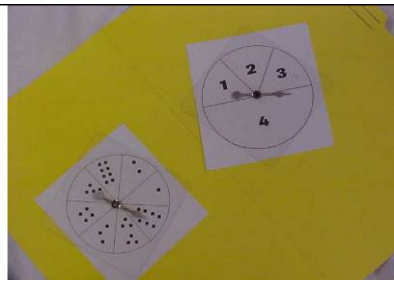
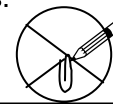
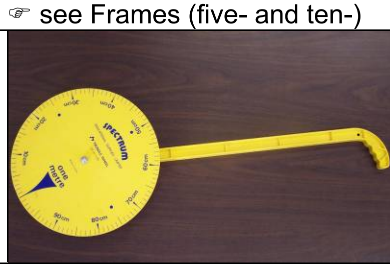
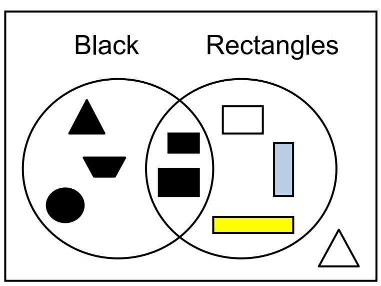
Sets typically include a variety of prisms, pyramids, cones, cylinders, and spheres.
The number of pieces in a set will vary.
Available in different materials (wood, plastic, foam) and different sizes.
</td></tr>
<tr><td>Geo-strips</td><td></td><td>

Plastic strips that can be fastened together with brass fasteners to form a variety of angles and geometric shapes.
Strips come in 5 different lengths. Each length is a different colour.
</td></tr>
<tr><td>Hundred Chart</td><td>
Name: Date:
Line Master 3 100-Chart
1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70
71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90
91 92 93 94 95 96 97 98 99 100</td><td>

10 × 10 grid filled in with numbers 1-100 or 0 - 99.
Available as a Black line Master in many resources or you can create your own.
Also available as wall charts or “Pocket” charts where cards with the numbers can be inserted or removed.
</td></tr>
</table>

Hundred Grid		• 10 × 10 grid. • Available as Black line Master in many resources.
Hundredths Circle		• Circle divided into tenths and hundredths. • Also known as “percent circles”.
Learning Carpet®	 http://www.thelearningcarpet.ca	• 10 × 10 grid printed on a floor rug that is six feet square. • Number cards and other accessories are available to use with the carpet.
Linking Cubes	☞ see Cubes (Linking)	
Mira®		• Clear red plastic with a bevelled edge that projects reflected image on the other side. • Other brand names include: Reflect-View and Math-Vu™.
Number Cubes	☞ see Dice (Number Cubes)	
Number Lines (standard, open, and double)		• Number lines can begin at 0 or extend in both directions. • Open number lines do not include pre-marked numbers or divisions. Students place these as needed. • Double number lines have numbers written above and below the line to show equivalence.
Open Arrays	☞ see Arrays and Open Arrays	
Open Number Lines	☞ see Number Lines (standard, open, and double)	
Pan Balance	☞ see Balance (pan or beam)	

Pattern Blocks		• Standard set includes: Yellow hexagons, red trapezoids, blue parallelograms, green triangles, orange squares, beige parallelograms. • Available in a variety of materials (wood, plastic, foam).
Pentominoes		• Set includes 12 unique polygons. • Each is composed of 5 squares which share at least one side. • Available in 2-D and 3-D in a variety of colours.
Polydrons		• Geometric pieces snap together to build various geometric solids as well as their nets. • Pieces are available in a variety of shapes, colours, and sizes: Equilateral triangles, isosceles triangles, right-angle triangles, squares, rectangles, pentagons, hexagons • Also available as Frameworks (open centres) that work with Polydrons and another brand called G-O-Frames™.
Power Polygons™		• Set includes the 6 basic pattern block shapes plus 9 related shapes. • Shapes are identified by letter and colour.
Rekenrek		• Counting frame that has 10 beads on each bar: 5 white and 5 red. • Available with different number of bars (1, 2, or 10).

Spinners		• Create your own or use manufactured ones that are available in a wide variety: o number of sections; o colours or numbers; o different size sections; o blank. • Simple and effective version can be made with a pencil held at the centre of the spinner with a paperclip as the part that spins. 
Tangrams		• Set of 7 shapes (commonly plastic): o 2 large right-angle triangles o 1 medium right-angle triangle o 2 small right-angle triangles o 1 parallelogram o 1 square • 7-pieces form a square as well as a number of other shapes. • Templates also available to make sets.
Ten-frames	☞ see Frames (five- and ten-)	
Trundle Wheel		• Tool for measuring longer distances. • Each revolution equals 1 metre usually noted with a click.
Two Colour Counters	☞ see Counters (two colour)	
Venn Diagram	 	• Used for classification of different attributes. • Can be one, two, or three circles depending on the number of attributes being considered. • Attributes that are common to each group are placed in the interlocking section. • Attributes that don't belong are placed outside of the circle(s), but inside the rectangle. • Be sure to draw a rectangle around the circle(s) to show the "universe" of all items being sorted. • Similar to a Carroll Diagram.

List of Grade 6 Specific Curriculum Outcomes

Number (N)

1. Demonstrate an understanding of place value for numbers: greater than one million; less than one thousandth.
2. Solve problems involving large numbers, using technology.
3. Demonstrate an understanding of factors and multiples by: determining multiples and factors of numbers less than 100 identifying prime and composite numbers; solving problems involving multiples.
4. Relate improper fractions to mixed numbers.
5. Demonstrate an understanding of ratio, concretely, pictorially and symbolically.
6. Demonstrate an understanding of percent, (limited to whole numbers) concretely, pictorially and symbolically.
7. Demonstrate an understanding of integers, concretely, pictorially and symbolically.
8. Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).
9. Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).

Patterns & Relations (PR)

(Patterns)

1. Demonstrate an understanding of the relationship within tables of values to solve problems.
2. Represent and describe patterns and relationships using graphs and tables.

(Variables and Equations)

3. Represent generalizations arising from number relationships using equations with letter variables.
4. Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.

Shape and Space (SS)

(Measurement)

1. Demonstrate an understanding of angles by: identifying examples of angles in the environment; classifying angles according to their measure; estimating the measure of angles using 45°, 90° and 180° as reference angles; determining angle measures in degrees; drawing and labelling angles when the measure is specified.
2. Demonstrate that the sum of interior angles is: 180° in a triangle; 360° in a quadrilateral.
3. Develop and apply a formula for determining the: perimeter of polygons; area of rectangles; volume of right rectangular prisms.

(3-D Objects and 2-D Shapes)

4. Construct and compare triangles, including: scalene; isosceles; equilateral; right; obtuse; and acute in different orientations.
5. Describe and compare the sides and angles of regular and irregular polygons.

(Transformations)

6. Perform a combination of translation(s), rotation(s) and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image.
7. Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations.
8. Identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs.
9. Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).

Statistics and Probability (SP)

(Data Analysis)

1. Create, label and interpret line graphs to draw conclusions.
2. Select, justify and use appropriate methods of collecting data, including: questionnaires; experiments; databases; electronic media.
3. Graph collected data and analyze the graph to solve problems.

(Chance and Uncertainty)

4. Demonstrate an understanding of probability by: identifying all possible outcomes of a probability experiment; differentiating between experimental and theoretical probability; determining the theoretical probability of outcomes in a probability experiment; determining the experimental probability of outcomes in a probability experiment; comparing experimental results with the theoretical probability for an experiment.

REFERENCES

Alberta Education. LearnAlberta.ca: Planning Guides K, 1, 4, and 7, 2005-2008.

American Association for the Advancement of Science [AAAS-Benchmarks]. *Benchmark for Science Literacy*. New York, NY: Oxford University Press, 1993.

Banks, J.A. and C.A.M. Banks. *Multicultural Education: Issues and Perspectives*. Boston: Allyn and Bacon, 1993.

Black, Paul and Dylan Wiliam. “Inside the Black Box: Raising Standards Through Classroom Assessment.” *Phi Delta Kappan*, 20, October 1998, pp.139-148.

British Columbia. Ministry of Education. *The Primary Program: A Framework for Teaching*, 2000.

Caine, Renate Numella and Geoffrey Caine. *Making Connections: Teaching and the Human Brain*. Menlo Park, CA: Addison-Wesley Publishing Company, 1991.

Computation, Calculators, and Common Sense. May 2005, NCTM.

Davies, Anne. *Making Classroom Assessment Work*. British Columbia: Classroom Connections International, Inc., 2000.

Hope, Jack A. et.al. *Mental Math in the Primary Grades* (p. v). Dale Seymour Publications, 1988.

National Council of Teachers of Mathematics (NCTM). Curriculum Focal Points for Prekindergarten through Grade 8: A Quest for Coherence. Reston, VA: NCTM, 2006.

_____, *Mathematics Assessment Sampler, Grades 3-5*. edited by Jane Reston, VA: NCTM, 2000.

_____, Principles and Standards for School Mathematics. Reston, VA: NCTM, 2000.

OECD Centre for Educational Research and Innovation. *Formative Assessment: Improving Learning in Secondary Classrooms*. Paris, France: Organization for Economic Co-operation and Development (OECD) Publishing, 2006.

Rubenstein, Rheta N. *Mental Mathematics beyond the Middle School: Why? What? How?* September 2001, Vol. 94, Issue 6, p. 442.

Shaw, J.M. and Cliatt, M.F.P. (1989). “Developing Measurement Sense.” In P.R. Trafton (Ed.), *New Directions for Elementary School Mathematics* (pp. 149–155). Reston, VA: National Council of Teachers of Mathematics.

Small, M. *Making Math Meaningful to Canadian Students, K-8*. Toronto: Nelson Education Ltd., 2008.

Steen, L.A. (ed.). *On the Shoulders of Giants – New Approaches to Numeracy.* Washington, DC: National Research Council, 1990.

Stenmark, Jean Kerr and William S. Bush, Editor. *Mathematics Assessment: A Practical Handbook for Grades 3-5*. Reston, VA: National Council of Teachers of Mathematics, Inc., 2001.

Van de Walle, John A. and Louann H. Lovin. *Teaching Student-Centered Mathematics, Grades K-3.* Boston: Pearson Education, Inc. 2006.

Van de Walle, John A. and Louann H. Lovin. *Teaching Student-Centered Mathematics, Grades 3-5.* Boston: Pearson Education, Inc. 2006.

Van de Walle, John A. and Louann H. Lovin. *Teaching Student-Centered Mathematics, Grades 5-8.* Boston: Pearson Education, Inc. 2006.

Western and Northern Canadian Protocol. Common Curriculum Framework for K-9 Mathematics, 2006.